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THE STRUCTURE OF ABRADED GLASS SURFACES

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SUMMARY. The physical nature of the processes involved in the workshop operations of cutting and grinding are but little understood, such attention as these operations have received being mainly of an empirical nature. In the present paper some preliminary observations are recorded on the tooling of hard brittle substances where the phenomena may be expected to be simplest. Even here they are sufficiently complex. The primary object is to substitute the conception of a flaw- or fissure-complex for the current view of a hill-and-hollow structure, as characterising a ground surface in brittle materials.

PART I. *Section I* describes observations on the flaws produced in glass by stationary, rolling and sliding spheres, and by glazier's diamonds and wheels.

Section II deals with the structure of ground glass surfaces, applying the results of the previous section, and showing the surface to be a fissure-complex.

Section III explains the double refraction observed by Twyman near the surface of these pieces of ground glass.

PART II. *Section I* shows the enhanced solubility of ground surfaces in hydrofluoric acid, and suggests a value for the upper limit of the thickness of the flowed, or surface-tension, layer on polished glass.

Section II describes the structure of polished surfaces of glass and similar materials as developed by etching, showing that mechanical abrasion is still active during polishing, and also confirming a low value for the thickness of the surface tension film.

PART I

SECTION I. THE CUTTING, ABRADING, AND POLISHING OF BRITTLE MATERIALS, WITH ESPECIAL REFERENCE TO GLASS

THE behaviour of a material when stressed beyond the elastic limit is a matter which appears to defy all attempts at mathematical analysis, and this is true not only of materials which enter on a ductile stage after the elastic limit is passed,

but also of brittle materials which are elastic up to the instant of fracture. The failure of the mathematical treatment in the simplest cases is much more complete than is generally realised. The brilliant analysis of Hertz on the contact of elastic solids has been shown by Auerbach to fail entirely to predict when failure will occur; it does not indicate with any exactness *where* failure will occur, and it is of course quite unable to indicate what the progress of events will be once an initial fracture has been produced. Where the particular analysis of Hertz fails, the much more general principle of similitude fares no better. There is, in fact, a complete discordance between theory and experiment which is overlooked by several recent writers.

In the present paper the experimental argument only will be pursued, and the theoretical treatment dealt with in a later paper.

The nature of the cracks produced when two spheres of a brittle substance are pressed into contact has been described by many writers. Most of them, with the exception of C. V. Raman, refer to the fracture as a cone, which, strictly speaking, it is not. The formation of the flaw (Fig. 1) is readily observed in the case of glass through a polished side of a specimen against the plane upper surface of which a steel bicycle ball is pressed. In the photograph, *AA* is the surface of the glass, the flaw being seen in elevation. The part above *AA* is, of course, the image of the flaw reflected in the polished surface of the glass.

As the load is increased, the contact circle increases until, at a certain critical value, the glass fractures near the edge of the area of contact. Without appreciable further increase of load the flaw spreads rapidly round the characteristic hyperbolic part of its course and then extends under a slowly increasing load along a sensibly conical sheet. It is seldom that the initial flaw forms with perfect symmetry round the axis of thrust. Frequently one side forms first and the flaw then, instead of conforming to the asymptotic cone, extends on one side only and becomes concave towards the thrust axis. This curling in may proceed to extraordinary lengths and will be discussed later in connection with rolling ball flaws.

The Progress of a Chattering Needle or Ball which does not rotate.

W. B. Hardy has found that, whenever two chemically clean surfaces are in contact and an attempt is made to slide one past the other, the material seizes. This, however, does not occur if the surfaces are not chemically clean. If the thrust be sufficient a flaw is formed, outcropping in an incomplete circle on the side of the ball in the acute angle between the direction of the thrust and the plane surface. The flaw penetrates into the glass in an incomplete hyperboloid sheet which has the same general form as that already mentioned. If the thrust be sufficiently oblique the ball or needle end moves across the surface, producing a sequence of such flaws. Fig. 2 shows a plan view of these flaws which will be described as chatter flaws*. The sequence of flaws regarded as an individual whole we shall describe as a chatter cut or chatter sleek. The appearance of these flaws in side elevation is shown in Fig. 3, where the upper half of the picture is the reflection of the lower half in the surface of the glass. The flaws are seen to have

* Similar photographs have been given by many writers.

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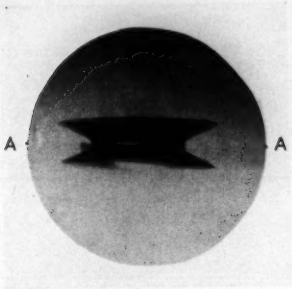

Fig. 1

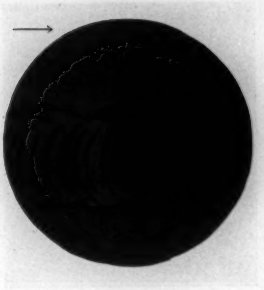

Fig. 2

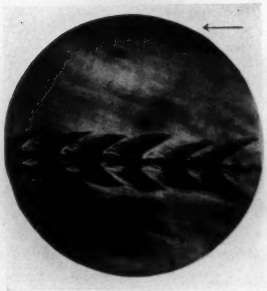

Fig. 3

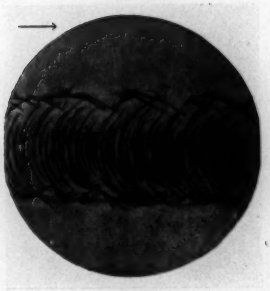

Fig. 4

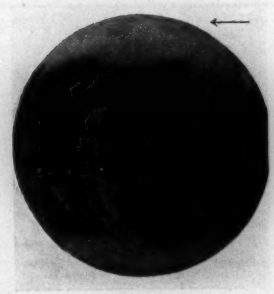

Fig. 5

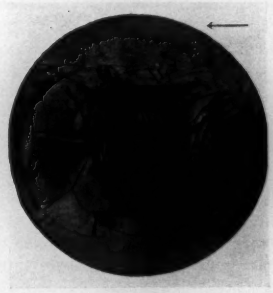

Fig. 6

a distinctly hyperbolic outline and occur at fairly regular intervals in the rear of the travelling particle. It will be noticed that though the glass has been much fractured by the passage of the needle no sensible amount of material has been actually detached.

The following points must be noted here:

(1) The complete sleek consists of three parts as seen in plan (Fig. 2), namely, the central series of arcuate chatter flaws and two lateral series of flaws arranged in cascades down the sides. The diameter across the cascades is the full width of the sleek, but it may happen that the cascades are not produced, or again the cascade flaws may appear without the central chatter flaws proper. The sleek, therefore, is capable of being dissected under circumstances which are not yet understood and the various parts of it may exist independently.

(2) A given ball cannot produce a sleek of less than a certain critical width. This critical width is practically the same as the diameter of the initial flaw-circle in the static example. The width of the sleek may be increased slightly by increasing the load, but the increase of width is limited to about 30 per cent. and is accompanied by a change in the character of the sleek. At the lower critical load the chatter flaws are at a relatively great distance apart and the cascade flaws approximate to straight lines representing the envelope of the chatter flaw sequence. An increased pressure widens the sleek, reduces the pitch of the chatter flaws, and develops cascades which are more arcuate and less continuous in character (Fig. 4).

For steel balls on glass the ratio of the breadth of the sleek to the pitch of the chatter flaws under critical loading is generally about 6. By using excessive loads this ratio may be increased to 12 or 13, but with still greater loads the glass begins to splinter and break away in fragments and the character of the sleek is then entirely lost.

Sleeks Produced by a Rolling Ball.

The sleek produced by a rolling ball resembles superficially that produced by a ball which does not rotate. It consists in plan view of a sequence of arcuate fractures but the arcs are reversed in direction; their concave sides face the direction from which the ball came. With a non-rotating ball the reverse is the case.

If the pressure is appreciably greater than the critical pressure there is a pronounced tendency to develop longitudinal flaws penetrating vertically into the glass near the median line (Fig. 5). No tendency to form these longitudinal flaws was observed in the case of non-rotating balls or spherical points. The rotating ball also differs from the sliding needle point in producing a less marked formation of cascade flaws (Figs. 5 and 6); the cascade flaws in fact are generally not visible without etching. After etching they appear to be merely continuations of the chatter flaws and do not seem to be capable of existing independently. Etching also not infrequently shows arcuate flaws facing in the same direction as those produced by a chattering needle. These flaws are not always present and are apparently of secondary importance. They stop sharply where they meet one of the primary "reversed" fractures (Fig. 7).

The asymmetric character of the flaw produced by a rolling ball under a heavy load is so marked that the flaw curls round and begins to approach the surface. Fig. 8 gives an idea of the outline of these flaws, but the complete outline can rarely be obtained in a photograph. The drawings in Fig. 9 give some idea of the shape of the complete flaw.

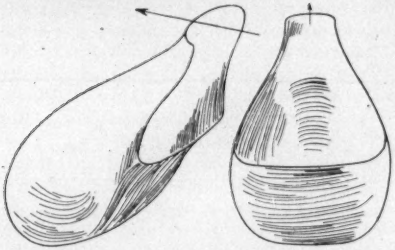

Fig. 9

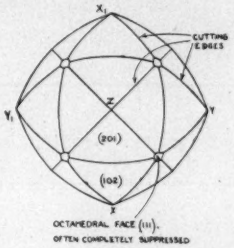

Fig. 10

The Action of the Glazier's Cutting Diamond.

The technical importance of the glazier's diamond is considerable, but so far as I have been able to ascertain its action has not been studied in detail. The diamond crystal belongs to the cubic system and the primary form seems to be the octahedron or possibly the tetrahedron. Fortunately the primary form is uncommon and generally secondary faces, such as those of the icositetrahedron are developed, these faces being generally curved*. The intersection of these faces gives an edge which has the profile of an axe, though of course it is very obtuse in cross section (Fig. 10). In use the diamond is so held that the pressure comes on this curved edge and care must be taken that it does not come on the point.

The structure of a glazier's cut consists of three longitudinal flaws, a median one and two lateral "subcutaneous" ones. These are shown in cross section in Fig. 11. Here *AA* is the original surface of the glass; *O* is the path of the diamond, *OB*, *OB* are the two lateral flaws and *OC* is the median flaw. This median flaw originally terminated at *C*, but the specimen was afterwards broken through. It will be noticed that the surface at *O* is hardly disturbed at all, but the glass above the right-hand flaw *OB* has flaked away.

Fig. 12 shows the appearance of the median flaw in elevation after the glass has been broken through. The direction of passage of the diamond was from left to right as shown and the curved marks on the surface have the same general outline as the profile of the chatter flaws in cross section. An interesting point is a second set of marks orthogonal to the first.

A plan view of a glazier's cut is shown in Fig. 13. It shows the narrow and shallow central groove made by the diamond and a number of fine cracks pointing backwards, which may be true chatter flaws analogous to those made by a sliding

* The curvature of the faces almost reduces the icositetrahedron to a rhombic dodecahedron in many of the examples which I have examined.

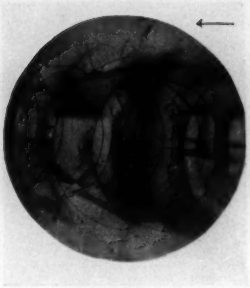

Fig. 7

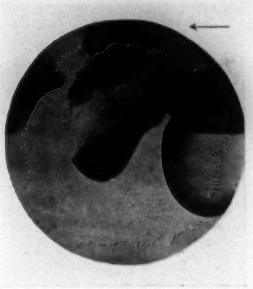

Fig. 8

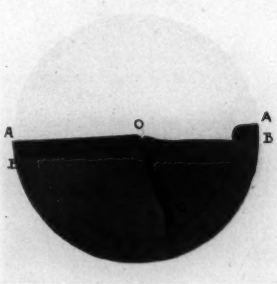

Fig. 11

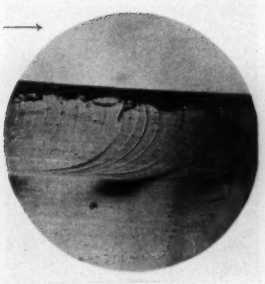

Fig. 12

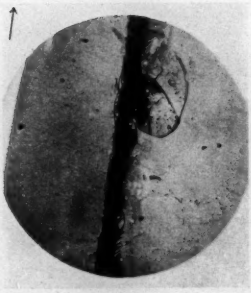

Fig. 13

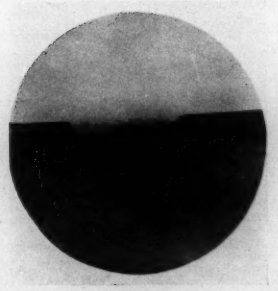

Fig. 14

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needle. The edges of the lateral flaws extend beyond the boundaries of the photograph, but on the extreme right there can be seen a number of parallel interference bands indicating the existence of the right-hand flaw.

Glazier's wheel cuts are in all respects similar but are not so perfect in formation. Fig. 14 shows the three flaws in cross section, the surface having flaked up on both sides. Fig. 15 is the elevation of the median flaw and shows the orthogonals very clearly.

The glazier, who of course requires the median flaw to be produced, uses a diamond which, by reason of its extreme hardness, maintains its keel edge and does not wear rapidly to a spherical form, or he uses a wheel which produces a median flaw partly by reason of its keel-shape and partly by virtue of its rotation.

It will be noticed that the types of flaws which can be produced in the glass are very varied but that, in general, the passage of a cutting or a flawing agent causes very little disturbance at the surface and removes very little material. The amount of material pushed away in front of the abrasive is extremely small and the main action is to produce a complex flaw structure beneath the surface of the glass.

In the light of these results we are able to discuss some of the properties of ground and polished surfaces of glass and other brittle materials.

Section II. Structure of "Grey" Surfaces

The traditional method of grinding glass and similar materials and the one which is invariably used to-day when the finest surfaces are required, is to lap them with a loose abrasive (e.g. sand or emery) trapped between a metal plate called the "tool" and the glass surface to be ground; as successively finer grades of abrasives are used the coarse irregularities are reduced step by step to a very fine stage. A limit is set to the degree of fineness obtainable by the tendency to seize and "cut" which becomes manifest in the later stages, and this limit unfortunately is such that the irregularities remaining are still somewhat greater than the wave length of visible light. It is generally held that the surface would appear polished if it were possible to grind a little finer, in which case there would be no need for the final polishing operation*. As it is, the object of good smoothing is to leave as little work as possible to be done in polishing, which is a relatively slow, difficult, and somewhat uncertain business.

A ground surface is technically known as a "greyed" one, and the grinding operation, when a fine finish is required, is known as "smoothing."

It has hitherto been the universal view that a grey surface consists essentially of hills and pits. This is the natural workshop view and it is also the view tacitly endorsed by the late Lord Rayleigh, by Dr J. W. French†, and other writers on the subject. It is the purpose of the present section to show that this does not give a complete interpretation of the true structure of the surface layers, in that it ignores entirely the condition of the glass immediately below the pitted surface.

The earliest experimenters (Hooke and Herschel) held the view that *polishing* (a subject with which we are not here so directly concerned, though it will come up for consideration in the section on Etch Patterns) was simply a continuation

* Lord Rayleigh, *Proc. Roy. Inst.* (1901), p. 563; *Proc. Opt. Conv.* (1905); *Collected Works*, vol. 4, p. 542.

† *Trans. Opt. Soc.* (Nov. 1916), etc.

of the grinding process; Lord Rayleigh (*loc. cit.*), however, indicated in 1901 that he suspected polishing of being a molecular or "nearly molecular" process, and in the next few years G. T. Beilby developed the view that a polished surface is essentially one which, having attained for an instant a liquid-like mobility, has come to rest under the action of surface-tension forces. Beilby shows that, though surface-tension and plastic flow effects were especially characteristic of ductile materials such as the metals, yet even in brittle substances (including glass) surface-tension effects were to be observed. Other experimenters, notably Osmond, have helped to illustrate this theory; but the concentration of attention during the last decade or so on surface-tension effects and plastic flow has tended to result in neglect of the phenomena of mechanical abrasion, whether in the case of ordinary grinding or in the case of polishing, where, although obscured by surface-tension effects, the forces of mechanical abrasion are still active.

We are here concerned, in the first instance, with "grey" surfaces in which surface-tension effects are not prominent. The depth of "grey" left to be removed in polishing may be estimated by several methods, which agree in making it very small in comparison with the usual dimensions used by the engineer.

The late Lord Rayleigh described two methods. The first consisted in observing the loss of weight during the polishing of a large flat disc. The surface was greyed fairly finely, and it was estimated that there was removed a depth of about six wave lengths for an incomplete polishing, "...but this would probably have to be increased to 10 or 12 wave lengths to get to the bottom of the deepest pits." The second method, which consists in smoothing a disc to a flat surface and polishing a hollow in it until the centre of the hollow is free from grey specks, gave a value of about three wave lengths for an incomplete polish.

This second test we have repeated on a number of occasions, and, though the values obtained are somewhat variable, it appears that a complete polish may generally be reached in about six wave lengths and a very fair polish in three or four. These values agree fairly well with Lord Rayleigh's. Direct microscopical examination carried out at the British Scientific Instrument Research Association has been found to give a value of about three wave lengths for the deepest pits left by fine smoothing.

There are two or three other methods, such as the observation of the limiting angle of specular reflection, which however give only a statistical average for the probable depth of the grey. These methods, though of a rather unsatisfactory character, agree in suggesting a value not much greater than one wave length as the depth of an average representative pit.

The general trend of the evidence, therefore, is to show that methods which involve the actual removal of the grey indicate that there is more work to be done than the methods involving mere inspection would suggest. The evidence is perhaps not sufficiently precise to do more than raise suspicions as to the nature of a ground glass surface.

We have reason to suspect, in view of what has been said in the previous section, that penetrating flaws may play a large part in the structure of abraded

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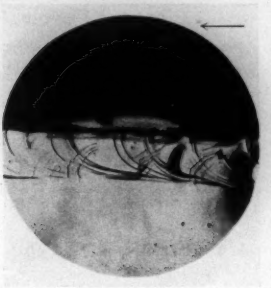

Fig. 15

Fig. 16

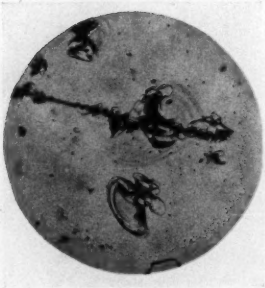

Fig. 17

Fig. 18

Fig. 19

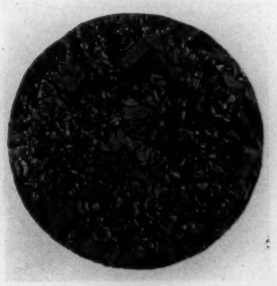

Fig. 20

glass. We have in fact seen that when conditions are at their simplest the passage of an abrading grain breaks up the surface layer of the glass to a considerable depth, though relatively little material is actually removed.

It is commonly supposed that an abrading grain acts by pushing away material that obstructs its passage, either by digging its nose into the glass and flaking out a conchoidal splinter, or by shearing away the top of an existing prominence. Without denying the possibility of such a method of action we can say that we have failed to observe evidence of its playing any part in the grinding process.

It has been shown in the previous section that the passage of a rounded grain produces a series of penetrating flaws (chatter-flaws) in its rear, unless it be a rolling grain, when it produces them in front. In either case it removes practically no material. An angular abrasive, such as a diamond, tends to produce penetrating flaws to the sides of its track; this results from the action of a moderately sharp *edge*. Similar cuts penetrating less deeply would be produced by a fine abrasive with clean crystal edges; the smaller penetration produced in this case is due to the smaller pressure exerted on each individual edge. In the case of an abrasive possessing sharp *points* we may have the nose digging into the glass and producing a percussion flaw somewhat similar to the flaws produced by a static load. If the abrasive is fixed in the tool it will then inevitably shatter the glass in front of it. If, however, the abrasive is loose, it will, in accordance with the principle of least work, swing round into some other position where its passage will meet with less resistance. We shall thus have a number of percussion flaws, but it is quite likely that even in such cases very little material will be removed. Removal of material in any large quantity will only begin when the various flaws have begun to intersect in a number of directions at the same part of the surface.

Fig. 16 shows what happens when a polished surface of glass is submitted to a new grinding operation with loose abrasive, or, as the workshops will term it, when the glass is "re-greyed." The figure shows an early stage in re-greying with sharp carborundum. We have a number of irregular percussion flaws made by points of the abrasive. The glass is badly shattered around the point of impact and then the grain of abrasive seems to have gone out of action for a time, later making another flaw further along. With this abrasive the tracks of the grains consist almost entirely of disconnected percussion flaws, and anything approaching a scratch or a "cut"* is rare.

Fig. 17, however, shows a cut made on the same specimen by the same abrasive. Here the glass has been flawed to each side of the track of the grain, recalling the action of the glazier's diamond, though, of course, the regularity is wanting. Fig. 18 shows the commencement of a re-grey with an abrasive prepared by the British Scientific Instrument Research Association (D_3). Here we get quite characteristic chatter cuts, and little else of importance. With some abrasives we have in addition cuts consisting essentially of a series of cascade flaws, and cuts occasionally approximating to the glazier's diamond type. Some examples of these various types will be seen in Figs. 2 and 16 to 19.

* Workshop term for an irregular scratch.

When a surface has been completely reground, so that no trace of the original polished surface is left, or when it is obtained by smoothing down from a more coarsely ground state, the direct microscopical examination is more difficult to carry out. There is a great deal of scattered light, and it is difficult to obtain at one and the same time a sufficient resolving power and a sufficient depth of focus to deal with so irregular a surface.

Photographs of grey surfaces, and of grey surfaces partly polished, have been given by many writers, but are quite uninstructive. Much more can be seen visually by continually changing the focus than can be photographed successfully, but even in visual work the structure is not of the clearest.

On etching the surface with hydrofluoric acid, however, the true nature of the surface is disclosed. The method was used by Lord Rayleigh*, but unfortunately he etched his specimens too much before examining them. A photograph of a long-etched grey surface is given in Fig. 23, which may be compared with Lord Rayleigh's in the paper mentioned. The genesis of this structure will be readily understood from Figs. 20 to 23, showing a 15-minute emery surface after etching for various lengths of time, and demonstrating that, contrary to Lord Rayleigh's interpretation, the hollows of Fig. 23 do not originate in pits but in *flaws*.

The elongated shape of the pits in the early stages of the etching shows that the original grey surface is dominated by a linear and not a point structure—by long narrow valleys and not by pits. But if these were simply valleys between more or less pyramidal hills, then the junctions of such valleys would virtually be pits, and after etching the identity of the hollows would be quite lost at the junctions. Actually we see that the hollows preserve their individuality very distinctly, and their ends are rounded off around the end of the original line-structure. Thus it is clear that we are not dealing with an ordinary valley structure at all, but the appearance of the etched surface is perfectly explained if the elongated hollows are regarded as widened flaws, developed by the acid according to the method of envelopes explained by Lord Rayleigh (*loc. cit.*).

It is clear from these observations that in a grey surface there are deep flaws extending far below the surface irregularities, but further evidence can be obtained by polishing a greyed surface with very fine abrasives.

It is not difficult to prepare chromic oxide whose effective grains are too small to produce any etch-pattern of sleeks; when a lens polished with such an abrasive is etched and examined with the dark ground condenser, the pattern is a remnant of the chatter grey from smoothing, resembling that of Fig. 37 in Section II of Part II of the paper, and consists of a chaos of unconnected penetrating flaws†.

* *Proc. Roy. Soc.* (1901); *Proc. Opt. Conv.* (1905); *Trans. Opt. Soc.* (1908); *Collected Works*, vol. 4, p. 542.

† It can be shown that the pattern is not due to the chromic oxide by first polishing the specimen with a good rouge and then following with the chromic oxide. The rouge polishes out the pattern left in the smoothing, cutting down to the bottom of the flaws produced in this process. When the rouge-polished surface is further polished with chromic oxide for varying periods the characteristic rouge-pattern is brought up on etching but it becomes less and less prominent according as the chromic oxide polishing is continued for longer and longer periods. After very long polishing with the chromic oxide the subsequent etching shows little of the rouge-pattern but under no conditions is there any reappearance of the chatter grey.

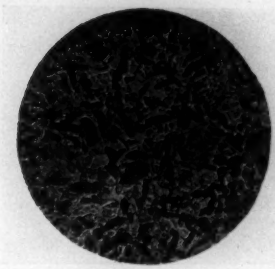

Fig. 21

Fig. 22

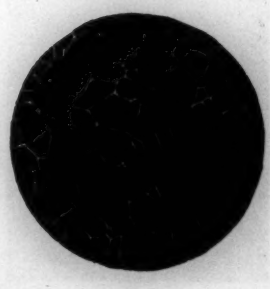

Fig. 23

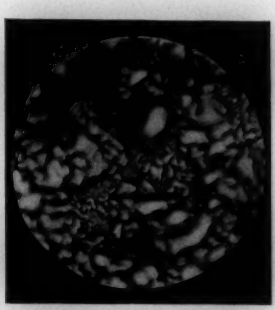

Fig. 24

Fig. 25

Fig. 26

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Confirmation by direct observation and measurement of the penetrating flaws and with the glass in its original ground state has been found possible by the staff of the British Scientific Instrument Research Association at Russell Square. For this purpose microscope cover-slips were ground on one side and examined with a 3 mm. apochromat oil-immersion lens of n.a. 1·4; the flaws were observed both through the grey surface and from the back, through the polished surface. Figs. 24 to 26 show at a magnification of 960 the appearance, as seen from the rear, of the same field focussed at various depths below the hill tops of the grey. It was found that the deepest pits extended to a depth of about three wave lengths, but the flaws to a depth three times as great. Something like a million flaws per square inch could be detected in some fields, but calculation from the indirect methods previously described shows that there are very many more than this in a finely ground surface.

The surface layer of a smoothed piece of glass consists, therefore, of a chatter flaw complex extending to a considerable distance below the actual surface; a grey surface, indeed, might be more accurately described as a "grey *layer*."

The mechanism of the ordinary process of smoothing, then, appears to be somewhat as follows:

The grains of abrasive are dragged about by the tool over the surface of the glass, producing chatter cuts or cuts approaching the diamond type, percussion flaws, cascade cuts, or intermediate forms of markings according to the character of the abrasive and the nature of its support.

Whatever the nature of the flaws so produced, practically no material is removed until several abrasive grains have passed over, or very nearly over, any given spot. When the flaws due to the several passages begin to intersect, material is removed, but only from the upper parts of the flaw complex; the flaws remain, extending much below the surface proper.

The flaws may be either of the diamond-cut type or may be chatter cuts such as are produced by a blunt needle passing over the surface. When the abrasive produces a flawing of the diamond-cut type it is probable that two or three passages will be sufficient to start the removal of material. On the other hand, in some observations made with a blunt needle in a shaping machine it was found that no material was removed until after four or five passages of the needle. The load here was rather more than critical; if it had been a minimum needed to produce a flaw at all, then probably ten or a dozen passages would have been required. (There is reason to believe that this is most often the condition obtaining in the polishing operation.)

When the material has been removed in this way and the surface is completely "grey," the surface irregularities—the hills and hollows—do not represent the full depth of the fractured layer. Below them a large number of flaws extend to a depth apparently two or three times as great. These flaws will in general be continuations of the sides of the "pits."

Since the greyed surface is thus broken up and full of narrow fissures, it will be easier to continue grinding on a surface already ground than it is to start a

grinding process on a polished surface. It is commonly observed in the workshop that a polished surface appears to have a tough "skin" and it has sometimes been thought that this is due to a surface layer tougher than the general body of the glass. It will be shown later that any such surface layer is too thin to affect a grinding operation with a coarse abrasive, and it appears more probable that the toughness is the true toughness of the massive glass and is due to the absence of large fissures such as are found in every "grey" surface.

The object of the smoothing process is not to splinter out fragments of glass by the frontal attack of the abrasive. It may happen at times that glass is sheared away in front of an advancing particle having a sharp edge or point, but such a process is difficult to keep under control and no evidence has been found of it in our observations. It is the nature of glass to fail in tension behind or to the side of the moving grain, and we rely on the intersection of flaws so produced to work down our brittle substances to the required form.

Section III. Structure of Glass Surfaces. The Twyman Effect

"If a very thin piece of glass, grey* on both sides, be polished so that it can be examined through its thickness and polarised light be applied, stress can distinctly be seen near each surface; if now one side be polished strain disappears from that surface, and the glass blows up; on polishing the remaining grey side, the glass becomes parallel again, and strain entirely disappears†."

The appearance of the glass as observed between crossed nicols is shown in Fig. 27, the appearance when examined through a Babinet compensator being as shown in Fig. 28. So far as the writer is aware, no explanation has hitherto been given of this phenomenon, but the simple stresses suggested in the foregoing paragraph represent a system which is not in equilibrium. The conditions that the strain may be self-contained in the body of the material are (1) that the total thrust across any complete plane section should be zero, and (2) that there shall be no resultant moment about any axis.

If we imagine that the fissures which have been shown to exist in a ground surface are slightly open at their mouths and are held open by minute fragments of glass or other materials, the system of stresses thus set up in the glass can be satisfactorily visualised. The surface would be in very severe compression for a depth of a few wave lengths, the material just below the grey layer would be in tension and this tension would diminish as we proceed away from the greyed surface, changing into a slight compression again as the other side of the glass is approached. The sum of the compressions in the greyed surface and in the other face of the glass will be equal to the tension existing in the material immediately below the grey layer and the system would be mechanically stable.

This view of the stresses is quite compatible with the appearance of the glass as shown between crossed nicols. The compression in the greyed surface itself cannot be detected visually on account of the minute depth of the chatter flaws.

* A glass surface which has been ground with a loose abrasive is technically known as a "grey" surface.

† F. Twyman, *Proc. Opt. Conv.* (1905), p. 78.

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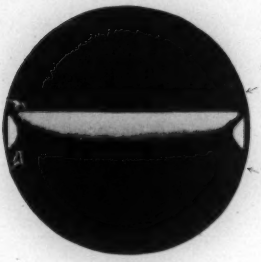


Fig. 27

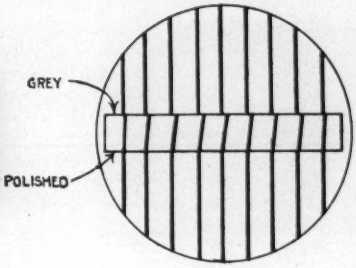


Fig. 28

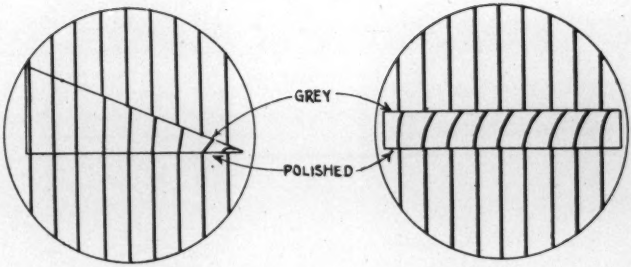


Fig. 29

Fig. 30

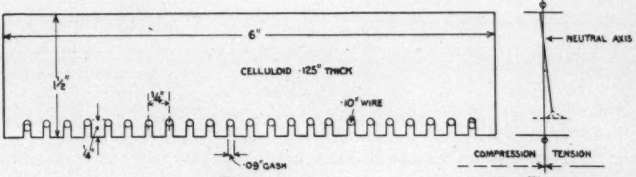


Fig. 31

Fig. 32

By means of a Babinet compensator it may be shown that the strain in the material below the grey layer is a tension, and this is the only strain producing a visible double refraction. The compression in the greyed surface and the tension immediately below the grey layer act along directions which are separated by a very small distance and the forces themselves are very nearly equal. The result of this is that these two forces constitute an almost perfect couple, but the moment of this couple is very small. The force required to keep the glass in equilibrium is the compression exerted on the polished side of the slab. This is distributed over an appreciable depth and acts at a considerable distance from the greyed surface. To produce equilibrium, therefore, this compression need only be quite small and its existence may easily be overlooked when the glass is examined in polarised light.

Confirmation of this view is obtained if a wedge-shaped specimen of glass be used, one side of the wedge being greyed and the other side polished. When this is examined in polarised light with a Babinet compensator the appearance is that shown in Fig. 29. (The specimen is an inch thick in the direction of the light path.) No effect can be observed until the specimen thins down to about ·15″ and the canting of the lines increases rapidly as the knife edge is approached. At the extreme limit the slope of the line becomes 90°. This would seem to imply that the action of the greyed surface is to impress a bending moment of constant amount on the material at each section, but as the section decreases in thickness the extreme stress produced by this constant bending moment increases, becoming infinite when the thickness of the material becomes infinitesimal.

The fact that the stress increases with increased coarseness of grey also confirms the view described above. To test the hypothesis experimentally an attempt was made to aggravate the Twyman effect as much as possible. One surface of a specimen of thickness ·15″ was greyed with the coarsest possible grey and the light was allowed to travel through three or four inches parallel to the greyed surface. When this specimen was examined in the Babinet compensator the appearance was that given in Fig. 30. The distribution of the strain parallel to the greyed surface and at right angles to the light path is one of a constant bending moment except near the ends where a slight modification takes place. This also is in agreement with theory.

The Twyman effect may be imitated by taking a piece of celluloid and cutting gashes along one side (see Fig. 31). Wires of slightly greater diameter than the width of the gashes are then pressed into the material as far as they will comfortably go. The resulting distribution of stress is found to be exactly analogous to that found in the Twyman effect, while the distortion of the Babinet bands is as in Fig. 32.

If then, in actuality, the Twyman effect be caused by minute particles of glass wedging open the mouths of the chatter flaws, it may be expected that a very little polishing will be enough to remove the effect, for the particles cannot enter far into the flaws. It is, in fact, found that the merest trace of polishing, consisting only of a surface gloss, removes the effect entirely. The grey is scarcely touched before the Twyman effect has completely disappeared. It may also be anticipated that immersing a specimen in a solution of hydrofluoric acid for a short time would

likewise cause the disappearance of stress; for the small particles that wedge open the flaws must be under very considerable compression, and strained materials are, in general, very much more susceptible to chemical action than material not so strained; also these particles are very small in comparison with their surface area. Experimentally it is found, in fact, that a very few moments' etching in a very dilute solution of hydrofluoric acid is enough to remove the effect almost completely. Some further observations relating to the etching away of the Twyman effect are given in Part II.

PART II

Section I. Observations on Etch Attack

Rate of Solution of various surfaces in hydrofluoric acid.

The experiments described in this section were undertaken in the first instance to provide further evidence on the Twyman effect, and also in some measure with a view to estimating the thickness of the flowed (surface-tension) layer on polished glass.

After a glass surface has been ground in the manner previously described, it is still necessary to polish it. This is done by means of a fairly fine abrasive* with a supporting material, capable of giving way to excessive pressures by elastic or plastic deformation. Materials commonly used in the elastic category are wood, cloth, felt, and in the plastic group, pitch and wax. The details of the process cannot be considered here. Brewster† nearly a hundred years ago gave an outline of methods which do not differ essentially from those in general use to-day, and no very important contributions to the literature of the subject, from the point of view of workshop practice, appear to have been made since. The best recent accounts are probably those of J. W. French in papers read to the Optical Society. Unless otherwise stated, the mention below of polished surfaces will refer to specimens polished on pitch.

As previously stated, Beilby‡ has demonstrated the existence on all polished surfaces of a flowed layer. On crystalline substances this layer is amorphous; on amorphous substances it is, of course, still amorphous, but somewhat different in its properties from the massive material below. Beilby showed that the flowed layer on calcite was practically confined to the region within 100 $\mu\mu$ ($\frac{1}{6}\lambda$) of the surface, though traces of mechanical disturbance made by the coarser particles could be detected to a depth of one to two wave lengths. On metals the flow is greater, but on glass and quartz we may reasonably expect that it will be less.

French§, however, has suggested that on glass there is a flowed ("beta") layer about eight wave lengths thick; the evidence is derived from a study of fire cracks. Polished specimens of glass were placed in a furnace at a temperature somewhat below the softening point; the surfaces developed a "crazed" appearance with

* The abrasives used for polishing glass may be much coarser than those employed with metals (*v.* below).

† *Edinburgh Encyclopædia* (1830), vol. 15, p. 660. Article on Optics.

‡ *Proc. Roy. Soc.* 82 A (1909), 599, etc.

§ *Trans. Opt. Soc.* (Nov. 1916).

numerous fine grooves or cracks on the surface. These cracks were rather uniformly about eight wave lengths deep; French assumes that this is the extent of the flowed layer, which has softened and contracted, while the massive material has not been permanently affected by the heat.

It seemed possible that some additional evidence might be obtained on this point by etching with hydrofluoric acid, and determining the rate of solution for material in the surface layer and for the material below the critical depth. If a change in the rate of attack were noticed when some eight wave lengths had been removed, this would strongly confirm the views of Dr French. If, however, no such change were found, this could not of itself be taken as negativing his views, because there may be no very great difference in the properties of the two materials. It has, we believe, been suggested that the flowed layer is more susceptible to attack by acid than the underlying material; we have therefore to look for a *decrease* in the rate of solution on passing below a depth of 8λ.

The value for the rate of solution of the massive material might also be obtained from observations of specimens obtained by fracture, if the area of their surfaces could be accurately determined; or from observations on specimens *ground* but not *polished*, if we did not suspect, in view of the Twyman effect, that there is something very abnormal about a ground surface.

In the section on the Twyman effect it was suggested that, if a greyed surface consisted of numerous minute flaws wedged open by particles of glass, attack by acid would be very rapid. We should expect this initial rapidity to fall off, however, within a short time, and to approach the value for the massive material as the Twyman effect disappeared. By observations of the progress of the acid attack on *polished* surfaces we may therefore hope to obtain some information on the depth of the flowed layer; and by observing the progress of the attack on *greyed* surfaces we may confirm our interpretation of the Twyman effect. The rate of attack will in each case ultimately approach that for the massive glass.

The specimens used were discs or cylinders of glass cut from the slab by the disc-cutting machine of British Patent 15,376/1915 (W. Taylor). In all the specimens a certain small proportion of the area exposed to the acid was a surface left by the machine, and it was therefore necessary also to determine the susceptibility of disc-cut surfaces to the attack of the acid.

(A) *Comparison of etch attack on disc-cut and greyed surfaces.*

The specimens were on the one hand a flat disc of 2″ diameter and ·05″ thick, and on the other two cylinders ·36″ in diameter and 1·46″ long, of the same glass (medium flint $\mu_D = 1{\cdot}604$, $\nu = 37{\cdot}8$, $\rho = 3{\cdot}47$). The grey surface was obtained by a moderate 15 min. emery smooth. They were etched for 25 seconds in a solution of HF with the following results:

	Area sq. cm.		Loss of weight gr.
Flat disc	grey	40·4	·0081
	disc-cut	2·0	
First cylinder	grey	1·33	·0010
	disc-cut	10·7	
Second cylinder	Same		·0008

These figures indicate that the susceptibility of the disc-cut surface is about ·3 of that of the greyed surface. Other independent experiments have given the ratio as about ·4.

(B) *Comparison of etch attack on greyed and polished surfaces.*

The specimens in this case were both discs of 2″ diameter. The greyed one was ·04″ thick, the polished one ·08″.

They were etched for 15 seconds in a dilute solution of hydrofluoric acid, with the results below:

		Area sq. cm.	Loss of weight gr.
First disc	disc-cut	1·6	·0088
	grey	40·4	
Second disc	disc-cut	3·2	·0010
	polished	40·4	

Using the value ·3 found in (A) above for the grey equivalent of a disc-cut surface, the figures here given indicate that the grey surface is attacked 11 times as quickly as the polished one. Other and independent experiments have given the ratio as about 12 or 13 to 1.

(C) The discs used in (B) were now alternately etched and weighed until the polished disc had lost about 15 wave lengths from each surface. The results are tabulated below.

Grey Disc				*Polished Disc*			
Time of etching	Weight gr.	Loss between weighings gr.	Loss per 15 secs. gr.	Time of etching	Weight gr.	Loss between weighings gr.	Loss per 15 secs. gr.
	8·2986				16·4430		
15 secs.		·0088	·0088	15 secs.		·0011	·0011
	8·2898				16·4419		
15 ,,		·0056	·0056	15 ,,		·0011	·0011
	8·2842				16·4408		
30 ,,		·0075	·0037	30 ,,		·0023	·0011
	8·2767				16·4385		
90 ,,		·0116	·0019*	90 ,,		·0055	·0009
	8·2651				16·4330		
15 ,,		·0022	·0022	15 ,,		·0008	·0008
	8·2629				16·4322		
15 ,,		·0019	·0019	15 ,,		·0008	·0008
	8·2610				16·4314		
Prolonged etching in another bath		—	—	Prolonged		—	—
	8·1977				16·3886		
15 secs		·0014	·0014	15 secs.		·0008	·0008
	8·1963				16·3878		
Long etching in another bath		—	—	Prolonged		—	—
	8·0943				16·3038		
15 secs.		·0008	·0008	15 secs.		·0009	·0009
	8·0935				16·3029		

* Owing to the two or three seconds that elapse before the acid can be washed off the specimen, a 90 seconds etching is not equal to six 15 seconds etching, so that this value is rather too low.

From these figures it would appear that the rapidity of the acid attack on the grey surface quickly declines, becoming ultimately the same as that on a polished

surface. The rate of attack on the polished surface remains constant, the slight decline observed being probably due to a change of temperature.

It seems that, if there is a layer of "beta" material extending to a depth of eight wave lengths, it does not differ in its solubility from the massive or "alpha" material.

The high initial values for the rate of attack on the grey surface can only be explained on the assumption that there is a relatively enormous surface area, 10 or 12 times that of the polished disc, or that the material in the surface differs radically from the massive material. The first alternative seems by itself to require altogether too high a figure, while we know from the Twyman effect that the surface material *is* in a radically different state, namely one of excessive strain. It seems probable that both effects may be present—a largely increased area due to chatter flaws, and a greatly enhanced solubility due to the strain. In etching, the strain rapidly disappears, reducing the acid attack to perhaps one-fourth or one-fifth of its initial value within two or three minutes, while the remaining excess of the rate above that on a polished surface may then be due to the greater area of the pitted surface.

(D) *Comparison of etch attack on coarse and fine greys.*

It has been found that the rates of attack on surfaces prepared by smoothing with 15-minute emery give very consistent results. In a comparison of the rates of attack on 15-minute and 60-minute emery smooths it was found that the latter was sensibly less affected at first—at roughly two-thirds of the rate of the coarser grey. This seems to agree with the polariscopic evidence that the Twyman effect is less with finer smoothing.

(E) *The depth of etching in the development of etch patterns.*

The specimens used for microscopic examination, as described in the section on Etch Patterns, were discs of the same glass as that of the above experiments, and ·65″ in diameter by ·1″ thick. A number of examples were etched and the loss of weight determined. It was found that the pattern was well developed in the matter of fine detail when the loss was ·0002 gramme, while the coarser structure could be best seen when the loss was about twice this. The area polished was 4·3 sq. cm., and the disc-cut area 1·32 sq. cm., equivalent to about 5 sq. cm. of polished surface. The loss per square centimetre of polished surface when the finer detail is best exposed is therefore ·00002 gramme or ·000006 cm. in depth (·0000023 inch—about one-tenth of a wave length).

It will be shown in the section referred to that in the etch patterns thus developed there is probably detail of the order of a few millionths of an inch, and beyond the limits of resolution of the microscope. This detail could hardly exist in a flowed (surface tension) layer, and it would therefore appear that the latter in the case of glass is only, at the most, some two or three millionths of an inch in thickness.

Section II. Etch Patterns

A surface which has been brought to a state of liquid-like mobility* and then, as it were, petrified in that condition, may be expected to be microscopically structureless; and as a matter of fact when a glass surface has been polished in the ordinary way and finished by dry polishing†, practically nothing can be seen of it in the microscope. But if the surface be lightly etched with dilute hydrofluoric acid‡, and suitable forms of illumination used, there is rendered visible quite a variety of etch patterns which we shall be able to interpret with the aid of what has been said in the previous sections.

Figs. 33, 34 and 38 show examples of the three chief types of polished surfaces. Fig. 33 represents the structure of a lens polished with rouge, the surface being covered with a great number of very fine sleeks of considerable length. The sleeks actually run in every direction indiscriminately and cover a very large part of the surface, but their structure is so fine that no illumination has been found that will enable all to be seen at once. It is necessary to use oblique illumination (in the present case transmitted by a dark ground condenser) and then those sleeks whose direction is at right angles to the light are rendered visible. Consequently the complete pattern with a dark ground condenser is a concentric one, somewhat as in Fig. 35, which we shall describe as a spider-web pattern.

Fig. 36 shows the same field as Fig. 33 with an illumination which shows that the heavier sleeks occur equally all over the field and run indiscriminately in all directions. But it is only in the illumination of Fig. 33 that the closeness of the web marks is really appreciated. It is a matter of great difficulty to photograph the web sleeks in a manner which shall satisfactorily bring out their extraordinary development, but Fig. 35 gives the best impression we have been able to obtain.

Fig. 34 shows a second type of pattern—extreme development of chatter sleeks. This specimen was worked on coarse manganese dioxide. Chatter sleeks are characteristic of polishing abrasives which contain large rounded particles; many commercial rouges fall in this category. If not too heavy, such sleeks are not harmful, as they are quite invisible before etching; but in the present example the chatter sleeks were so numerous and so pronounced that complete polishing was impossible.

Figs. 37 and 38 show examples of the third type of etch pattern, the "chatter grey" pattern. Chatter grey is the characteristic pattern left by fine chromic oxide, and other abrasives whose effective grains are very small, say of the order of $\frac{1}{4}\lambda$ in diameter. The three types of pattern are not mutually exclusive, and web sleeks, chatter sleeks, and grey may occur together in the same specimen.

The *chatter sleek* in its simplest form consists of a regular series of arc-shaped flaws; the sleek is therefore incomplete in the light of the section on the Abrading of Glass, in that the lateral "cascades" are missing. Many fairly regular chatter

* "The particles of rouge seem to have the power of seizing the surface so as to set the molecules gliding." Beilby, *Proc. Roy. Soc.* 72A (1903), 218.

† Technically known as "drying up the wet."

‡ A convenient strength is 2N.

Fig. 33

Fig. 34

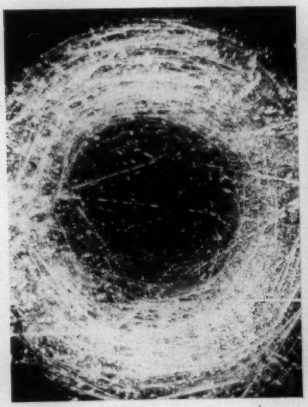


Fig. 35

Fig. 36

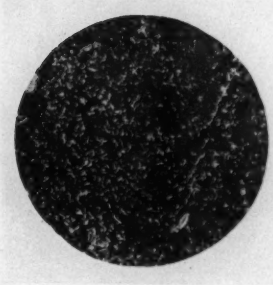


Fig. 37

Fig. 38

Fig. 39

sleeks are shown in Fig. 34 (manganese dioxide polish), but chatter sleeks are more often interrupted, so that blank spaces are left in the track of the grain (cf. Fig. 39). Occasionally there is a periodic and rapid increase and decrease of the width of the sleek without complete discontinuity, but more often a sleek which shows a periodic fluctuation of width is interrupted at the narrow points.

Web sleeks also are a product of the polishing operation. Examined by reflected light or with insufficient resolving power, they appear to be fine grooves in the surface of the glass. With transmitted dark ground illumination and sufficient power many such sleeks can, however, be resolved into chatter sleeks; they differ from the larger sleeks mainly in being less interrupted. Down to the limits of resolution of the microscope, that is, until the chatter flaws are closer together than a hundred thousand or so to the inch, it is possible to find chatter sleeks in increasing numbers. It seems very probable that web sleeks which are not thus resolvable are nevertheless chatter sleeks in constitution, but with detail of ultramicroscopic dimensions*.

In the ordinary way of polishing, as stated above, the sleeks run indiscriminately in all directions, but it is possible to prepare specimens in which the particles of abrasive have all passed across the surface in one direction only. When this is done it is found that in all resolvable sleeks the convex side of the flaw faces the direction from which the particle came. It follows, as shown in the first part of the paper, that all the effective particles in the polisher are *sliding* and not *rolling*.

Chatter grey is not a product of the polishing operation at all, but a remnant of the smoothing structure.

Besides the three main types above described we get asymmetric or twisted sleeks, of which examples can be seen in Fig. 39, and a few in Fig. 34. These are clearly the *cascade* flaws occurring without the chatter flaws.

There is a good deal of variety in the patterns falling under each of the above categories, and especially in those of the chatter sleek group. It is often possible to recognise a certain individuality in the sleeks and to detect several passages of the same grain across the surface. There may also be observed at times the evidence of the gradual breaking down of a large grain into two or three smaller ones, but as this process requires more than the width of the field of the microscope, it cannot readily be photographed.

The pitch of the chatter flaws in a very coarse chatter sleek made by rouge or other workable abrasive on ordinary glass is from 1/10,000 to 1/20,000 of an inch. A typical example (made by a commercial workshop rouge) was found to have chatters 24,000 to the inch. The width of the sleek was $1\frac{1}{2}$ ten-thousandths of an inch (7λ).

The detail of most web sleeks is beyond the limits of resolution of the microscope†, but by the aid of the diffraction pattern it has been possible on many occasions to assure ourselves that sleeks which were at first classed as web sleeks,

* This interpretation is of interest, inasmuch as it has hitherto been supposed that sleeks are fine grooves in the glass.

† It is a difficulty that the highest power objectives, being immersion lenses, are useless for this work.

and supposed to be no more than continuous fine grooves in the surface of the glass, were in reality chatter sleeks composed of a very regular sequence of chatter flaws at the rate of a hundred thousand or more to the inch.

There remain, however, the great body of true web sleeks in which no structure whatever can be made out; the width of many of these sleeks is undoubtedly below $\frac{1}{2}\lambda$, possibly considerably below, and only intense oblique illumination on a dark field renders them visible at all. The conclusion, however, seems to be irresistible that these also are chatter sleeks, and that their detail is of the order 10^{-6} of an inch. Now the atoms themselves are of the order 10^{-8} and if we can speak of molecules of glass, these must be approaching the order 10^{-7}, so that the chatter flaws of a web sleek may be said to occur at intervals of a few molecules. It seems to follow that any flowing of the surface layer (under the action of surface-tension, etc.) must, in the case of such materials as glass and quartz, be of extremely limited extent. Indeed, it seems very apparent that liquefaction of the surface layer to a depth of a few millionths of an inch would destroy all the detail of the finer sleeks.

Now the chatter flaw character of sleeks has been shown to obtain from the coarsest dimensions down to the limits of resolution of the microscope. It will be shown later* that the sleeks whose flaws are of the order 10^{-5} or 10^{-6} apart are caused typically by grains of diameter about 12 to 16 $\times$ 10^{-5} of an inch. But some abrasives that will polish, though extremely slowly, seem to have no grains of a stable nature above 10^{-5} of an inch. These abrasives leave no sleeks at all, so far as the microscopic evidence goes, though it is conceivable that they really produce chatter flaws of completely ultramicroscopic character (presumably of the order 10^{-7} of an inch apart by analogy with the larger grains). But the atoms themselves are not much smaller than this, and if we are to allow any thickness at all for the Beilby layer, it seems probable that the flaws of these smaller ultramicroscopic sleeks must be flowed together.

If we confine our attention to polishing abrasives and polished surfaces of a workshop character, it appears that the best abrasives are those that produce a good web pattern, the detail of the sleeks being just on the limits of resolution of the microscope (10^{-5} of an inch).

It remains to decide whether, under workshop conditions, the removal of glass is a truly molecular or merely a "nearly molecular" process. It is possible to induce a polish on glass without the help of any abrasive. In such a case the action is presumably molecular; glass is removed and may be detected later in the surface of the polisher†. The progress of the polishing in such conditions is very slow, and much greater rapidity may be attained by the use of abrasives. The speed of polishing in general increases with the coarseness of the abrasive, until the sleeks produced thereby become altogether too prominent. It seems difficult to regard the sleeks as a purely accidental feature, having no direct influence on the polishing of the glass; for we have seen that the best polishing abrasives are those that produce

* Paper to follow.
† J. W. French, *Trans. Opt. Soc.* 21 (1919–20), 81.

the finest pattern of sleeks, and the surface of the glass under these conditions is a mass of tiny flaws.

The process of polishing (on pitch polishers) seems therefore to be carried out in this way:

When the abrasive is first applied to the work, the particles are carried in a thin film over the whole surface of the glass, producing (in the case of a fine-grained spherical abrasive) innumerable chatter sleeks whose chatters are of ultramicroscopic, or nearly ultramicroscopic, dimensions. These sleeks, owing to the more spherical shape of the polishing abrasive, are more regular than those produced in "smoothing." Intersecting in all directions they break up the surface into a chatter flaw complex, and then in the dry polishing stage the polisher comes into intimate contact with the work and incipient cohesion takes place between polisher and work. At this stage the effort required to drive the polisher increases enormously, and the whole flaw-complex is broken away by the roots*. At the same time a slight molecular rearrangement of the new surface takes place, as observed by Beilby and others.

In the case of our own workshops, it is known that there is commonly removed in the polishing process from three to six wave lengths of thickness. It is found that with automatic polishing and when the polisher is working at its best, this amount is removed in some 50 or 60 "wets" (alternations of wet and dry polishing); consequently the average amount removed per wet is in the neighbourhood of one or two millionths of an inch. This it will be noted agrees with the general postulate, that the detail of an average spider-web is of the order of a few millionths of an inch. This interpretation, though in no way contradicting the experiments of Beilby, represents in some measure a return to the views of Hooke and Herschel; it raises a number of questions, some of which are capable of formulation in quantitative terms, and it is hoped to deal with some of them in a further communication. A considerable amount of experimental data has been collected, but its discussion involves a more detailed consideration of workshop practice and technique, which would increase unduly the length of the present paper.

It could not be observed that various types of glass differed at all in their etch patterns, while the structure of polished examples of quartz and of felspar was also very similar to that of polished glass. The crystal specimens (cut parallel to the long axis) show no influence, on the pattern, of the anisotropic character of the crystals. This is equivalent of course to saying that the crystals break equally well in any direction, or are devoid of any signs of cleavage. It was observed that felspar and quartz were much more resistant to hydrofluoric acid than glass. The structure of a glass surface was well developed by etching for 15 seconds in 2N hydrofluoric acid at 15° C. The felspar required five minutes to bring out the pattern, while the quartz required seven minutes with acid of 5N strength†.

* The same thing happens in time and in a more gradual manner even if the polisher is not allowed to dry.

† It is to be expected that various types of glass will etch at various rates, but no great difference was observed. The types used included Dense Flints, Hard Crowns, Barium Crowns, and plate glass.

The structure on apatite and fluorite appeared to be quite different. The etching agent was sulphuric acid, and the sleeks when developed did not resemble chatter sleeks, but rather the grooves produced on metals by polishing powders. This would suggest a certain plasticity in these crystals, which are of a mineralogically lower hardness (fluorite 4, apatite 5, felspar 6, quartz 7 on Moh's scale).

A Further Note on the Extent of Surface Flow in Glass

Beilby* has described the covering over of minute pits in metals by the smeared surface-tension layer. His observations relate more particularly to very ductile materials (copper) and the covering film is estimated to be 10 to 20 micro-millimetres in thickness (less than a millionth of an inch). French was unable to obtain any covered over cavities in glass; he experimented with a special glass containing many minute air-bells, which, once broken into, could not be smeared over. He also deposited a fine layer of silver on a grey surface and found that the silver was never covered in during the polishing process. He therefore concludes that the surface flow in glass is much less than in metals, and that it differs therefrom in the fact that the material must be ploughed away to the level of the bottom of the deepest pits.

The workman nevertheless believes that "grey" can be covered over, the curious workshop expression being that grey can be "*blinded in*." The workshop view is confirmed by the microscopic examination of etched surfaces, where chatter grey (a remnant of the smoothing structure) can be observed on polished surfaces of glass and quartz, which previously to etching showed no such structure. The workshop view does not contradict that of Dr French, when we interpret grey to mean "chatter grey," that is fissures rather than pits. The surface irregularities must be ploughed down to the level of the deepest pits in the polishing operation, but the whole surface is not necessarily ploughed away to the level of the deepest fissures.

A fine layer of silver can be deposited on the surface of the glass, but is quite unable to penetrate into the chatter flaws. Consequently the silver is all removed in the earlier stages of polishing, but the deep flaws still remain. If the surface layer be not too much broken up, these fractures may be covered in, but there is apparently a tendency for the severe tangential drag of the polisher to dislodge fragments that have been rendered insecure but not actually broken out in smoothing, so that it is also quite possible to "open-up" grey in polishing as one works downwards. The experiment with a silver deposit has been repeated, and a similar one made with iodeosin. Both the silver and the iodeosin stain disappeared at a very early stage, long before the grey. It seems clear that they only affected the surface irregularities and never penetrated the chatter flaws at all.

The results of a number of experiments seem to indicate further that but little covering over of the "grey" occurs with those coarse abrasives which are suitable for ordinary workshop practice; with such materials the tangential drag or "bite"

* *Proc. Roy. Soc.* 89 A (1914), 593; also 72 A (1903), 218.

is very considerable and the rouge grinds its way down till the bottoms of the deepest flaws are removed. On the other hand, with certain abrasives of very fine texture where the drag is negligible, a flowing of the surface layer seems to take place which results in covering over the last traces of the grey-complex.

I wish to acknowledge the valuable criticism of Sir Herbert Jackson and his staff of the British Scientific Instrument Research Association in the preparation of this paper.

DISCUSSION

Mr H. Moore said that the papers which the author submitted to the British Scientific Instrument Research Association had been passed on to him for criticism. A number of experiments had been made to test Mr Preston's rather startling conclusions. By using an oil immersion objective it had been possible to obtain photographic evidence of the presence of fissures well below the deepest holes left in abraded glass surfaces. The maximum depth to which the fissures extended was about 12λ. The general idea that an abrasive appeared to act like a plane over wood, the plane iron being loose, seemed to be entirely inadequate to explain the structure of the greyed surface obtained. The fissures under the greyed surface could only be explained on the lines suggested by the author.

It was Sir Herbert Jackson's view that the present paper constituted a marked advance in the subject. Apart from its scientific value, it should be of great assistance to manufacturers.

Dr J. S. Anderson asked if an interference method had been used to determine the depths of the fissures in terms of wave length.

Mr Moore explained that the microscope was fitted with a fine adjustment which had been checked in various ways; the results could be relied on to $\frac{1}{4}\lambda$.

Mr J. Guild said that the author's results appeared to give a reasonable explanation of the fact that the surface of a lens or prism, no matter how well polished it is, always scatters a good deal of light. The subcutaneous fissures, though too small to be seen, might be quite effective as scatterers of light.

Mr J. Rheinberg thought that the paper threw light on other problems. In platinising glass by heat processes the platinum seemed to penetrate only to a small extent—about $\frac{1}{8}$ or $\frac{1}{4}\lambda$ according to Beilby. The author had used deposits of silver. He suggested that platinum and palladium should be tried, as they were the only metals which he could get to enter the glass.

He had found that various types of glass etch at different rates.

Mr H. S. Ryland said that the existence below a polished glass surface of chatter sleeks and flaws seems to suggest an explanation of the "film" or "scum" which was so troublesome in enclosed optical instruments during the war. Grease, oil from pitch, etc., might be absorbed in minute quantities by these flaws, etc.,

and afterwards exude, forming the scum. This would appear to be borne out by the method of preventing scum which he had advocated before the Society; this briefly consisted in heating the lens or prism in hot water, and afterwards using suitable precautions to prevent foreign matter from reaching the surface.

He asked whether the extent of the chatter sleeks varied with the speed and pressure of the rolling or sliding ball. It was possible that they would increase with speed and pressure up to a point, falling off with any further increase. If this is so it would seem that in lens manufacture there is a critical speed and pressure at which the time needed for polishing and smoothing is a minimum.

It would be of interest to observe the structure, if any, of a "fire polished" surface after etching and to compare the structure, after etching, of surfaces ground on a wheel with that of those ground in the usual way with loose abrasive.

Mr P. F. Everitt thought that there was not a single phenomenon in connection with the process of grinding and polishing that the author had not explained.

In cutting glass with a diamond it was important to finish the process right out. In one case an operator had been told to cut 100 specimens and then break the edges, with the result that they all went to pieces. He suggested that the author might investigate the time factor.

Mr F. C. Bullock said that the effect of varying the speed of the rolling or sliding ball had not been tried, but with varying pressure the width of the sleek had been found to be a function of the diameter of the ball.

Author's reply (communicated): I am afraid that, notwithstanding Mr Everitt's kind remarks, we are still very far from understanding everything that goes on in the workshop in abrading glass. Workshop processes are in general most complex, and the tooling of any material, brittle or otherwise, is a matter not yet brought within the frontiers of Science.

With regard to Mr Guild's remarks, I had not thought of the ultra-microscopic fissure-complex as a possible cause of scattering of light; I have no observations on this point.

In reply to Mr Ryland, the structure of the "fire polished" surfaces is, in all the cases I have examined, radically different from that of a mechanically polished one. The structure of surfaces ground with wheels is generally fundamentally the same as that of one ground with loose abrasive, but wheel-ground surfaces present many features of special interest that are best reserved for a separate paper. The effect of varying the pressure on the appearance of the chatter sleeks is outlined in the paper; the speeds were not varied a great deal, but when expressed as diameters per second were of course very low compared with the speeds obtaining with rough grains in polishing. Expressed as an ordinary velocity (e.g. cm./sec.) the speeds used for the needle and ball would be comparable with the speeds commonly used in polishing optical glass.

In reply to Mr Everitt, I am aware that the glazier who is cutting glass with a diamond regards it as important to break the glass "while the cut is hot." If the cut is allowed to become cold the glazier says that it degenerates into a scratch.

The material above the subcutaneous (lateral) flaws flakes away in the interval, but what happens to the central flaw to prevent its extension on bending the glass. I am unable at present to say.

No particular importance is attached to the experiments in depositing silver on greyed surfaces, and it scarcely seems worth while to pursue the matter further for the present purpose. Mr Rheinberg's suggestion that in other problems the point may be of direct importance is interesting, and had not occurred to me previously. I am afraid that at present I shall not be able to investigate these matters, but hope that Mr Rheinberg may perhaps be able to do so.

DESCRIPTION OF FIGURES

Fig. 1. The hyperbolic (static) flaw in elevation. The upper part is a reflection of the lower, seen in the totally reflecting surface *AA* of the glass. ×48.

Fig. 2. The chatter sleek of a sliding needle, in plan. Needle end part of a 1-inch sphere. Load just critical. ×9.

Fig. 3. A similar sleek in longitudinal elevation, showing the chatter flaws. The upper half is again a reflexion of the lower. ×60.

Fig. 4. A sliding-needle sleek with excessive load, in plan. ×9.

Fig. 5. Plan view of a rolling ball's track. (Diameter of ball ½-inch.) ×60.

Fig. 6. A similar track after etching with HF. ×60.

Fig. 7. A similar track, showing secondary flaws. ×60.

Fig. 8. Side elevation of a rolling-ball track with heavy load. ×60.

Fig. 9. Apparent form of rolling-ball fractures, when complete.

Fig. 10. Form of the glazier's diamond crystal.

Fig. 11. Cross section of a glazier's diamond cut. ×36.

Fig. 12. Longitudinal section of same, broken through the central flaw. ×60.

Fig. 13. Plan view of a diamond cut, etched with HF. ×60.

Fig. 14. Cross section of a glazier's wheel cut. ×36.

Fig. 15. Longitudinal section of same. Note the orthogonal systems of markings, as in Fig. 12. ×36.

Fig. 16. Percussion flaws at commencement of a re-grey with coarse carborundum. (The focus is slightly below the surface of the glass and small surface markings appear as spots over the field.) ×250.

Fig. 17. A splinter cut and percussion flaws made by carborundum at commencement of a re-grey. The important flaws are to the *sides* of the cut; there are none at either *end*. The most important flaws are well below the surface and may be detected by the interference bands. ×60.

Fig. 18. Chatter cuts formed at the commencement of a re-grey with a soft variety of corundum (D_3). ×250.

Fig. 19. A very complex family of chatter cuts from (D_3) corundum. ×250.

Fig. 20. A plate glass surface smoothed with "15-minute" emery and etched for one minute with 3N hydrofluoric acid. ×250.

Fig. 21. The same after etching for 2 minutes. ×250.

Fig. 22. The same after etching for 7 minutes. ×250.

Fig. 23. The same after etching for 25 minutes. ×250.

The photographs in Figs. 24 to 26 were obtained by the staff of the British Scientific Instrument Research Association, Russell Square, from a cover glass ground on one surface with (C_2) corundum and photographed through the polished side using a 3 mm. apochromatic object glass of 1·4 n.a. × 960.

Fig. 24. Top of ground surface in focus.

Fig. 25. Bottom of pits in focus 3 wave lengths below 14.

Fig. 26. Shows flaw 10 wave lengths below 14.

It will be noticed in Fig. 25 that a number of flaws are also visible, some, of course, out of focus.

Fig. 27. Twyman effect between crossed nicols.

Fig. 28. Twyman effect in Babinet polariscope.

Fig. 29. Twyman effect on wedge of glass.

Fig. 30. Twyman effect with high resolution.

Fig. 31. "Artificial" Twyman effect specimen.

Fig. 32. Distribution of stress in above.

Fig. 33. The "spider-web" pattern, the fine sleeks developed by etching with HF.

Fig. 34. Chatter sleek pattern of coarse manganese dioxide.

Fig. 35. The complete web pattern.

Fig. 36. The same field as Fig. 33 in a different illumination, showing that the sleeks run in all directions.

Fig. 37. Chatter grey left, showing ground-mass of chatter flaw complex.

Fig. 38. Chatter grey from coarse emery, after partial polishing. Also a heavy chatter cut.

Fig. 39. Specimen polished with chromic oxide with help of rouge in polisher. Note the interrupted and twisted (cascade) sleeks. There is also a good spider-web and some chatter grey not visible in this lighting.

Magnification in Figs. 33–39, approximately 80 diameters.

THE STRESS CONDITIONS SURROUNDING A DIAMOND CUT IN GLASS

By A. J. DALLADAY, A.Inst.P. and F. TWYMAN, F.Inst.P.

MS. received, 14th November, 1921. Read and discussed, 9th February, 1922.

In connection with a certain manufacturing problem it was found desirable to examine the distribution of stress in the neighbourhood of a diamond cut in glass. The block of glass examined was of Chance's Hard Crown, Melt No. 3117*. It was rectangular in shape, 90 mm. × 35 mm. × 16 mm. The diamond cut was made along the 35 mm. dimension, and observation was made along its length. The total length of the cut was 30 mm. and it was a rather splintery one.

The block was first placed between crossed nicols, the plane of polarisation being vertical. The resulting pattern is shown in Fig. 1.

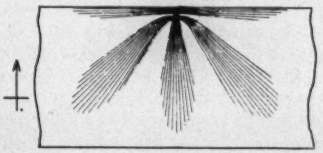


Fig. 1

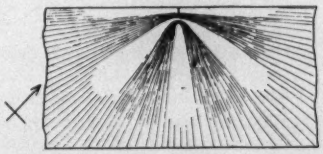


Fig. 2

When the plane of polarisation is at 45° with the vertical, the pattern is as shown in Fig. 2. The centre lines of the black bands in each case are isoclinic lines, or lines at every point of which the principal stresses have the same direction, i.e. 0° and 90° in Fig. 1, and 45° and 135° in Fig. 2. If, now, we draw two series of lines of which each intersects these isoclinic lines at the correct angles (viz. in the plane of polarisation and at right angles to that plane respectively), we shall obtain two orthogonal series of curves, the lines representing the directions of the principal stresses.

In Fig. 3 (*a*) these two sets of curves have been drawn for the 2 mm. square on one side of the vertical through the diamond cut. The conditions on the other side are exactly symmetrical. By examination at the free surface it was found that the lines shown thus - - - - are lines of compression, and the lines shown thus —··—··, therefore, of tension. The full lines are the centres of the black bands between crossed nicols at 90° and 45° respectively as marked on the diagram, and the crosses at intervals along them show the directions of principal stress.

The flaw shown in the diagram is characteristic, both in position and direction,

* $\mu_D = 1{\cdot}5166$, $\nu = 60{\cdot}9$.

and is to scale, as is the diamond fissure so far as is possible, considering the minuteness of the whole thing.

The essential features of the stress system seem to be a region of violent horizontal tension immediately below the diamond cut, less than $\frac{1}{10}$ mm. in depth, which changes abruptly to compression as we go further into the block, this compression diminishing rapidly, until at 2–3 mm. depth it is very slight indeed. The region of tension is so minute that it is impossible to form any estimate of the amount of tension. Some notion of the concentration of force at the bottom of the fissure can, however, be formed when it is remembered that the closeness of lines of compression and of tension is a measure of intensity when the same lines are followed to different parts of the diagram. The retardation observed at about 2 millimetres from the cut is about $\frac{1}{10}$.

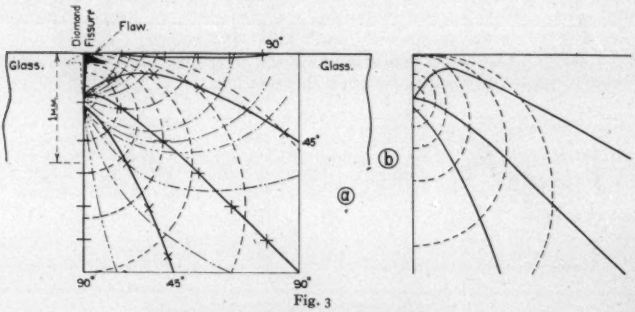


Fig. 3

Isoclinic lines of stress ——————

Direction of compression — — — — — —

" " tension —— · · —— · · ——

The flaw shown branching off from the diamond fissure would appear to be the result of the concentration of the lines of compression close to the surface. In a diamond cut which has not flawed it seems likely that practically all the compression effect is localised in the first $\frac{1}{10}$ mm. from the surface, for the freer the cut is from flaws the more arched does the top 45° isoclinic become, the direct effect of which is just this concentration of compression at the surface. Fig. 3 (*b*) illustrates this. As already stated, it is difficult to obtain an unflawed diamond cut of this character, and the effect of the flaw is partially to relieve the stress.

By heating the glass to a temperature 60° or 70° below the optical annealing temperature, at which former temperature it will attain 95 per cent. annealing in some three-quarters of an hour, practically the whole of the stresses may be made to disappear without disturbing the annealing of the block itself.

DISCUSSION

Professor E. G. Coker said that the paper was of great interest, and especially so to engineers who were continually finding that flaws and scratches exercised a very great influence on the strength of metals, and particularly of hard and brittle metals.

The case investigated bore some resemblance to some others he had examined in transparent materials, using nitro-cellulose models. If, for example, a semi-circular cut is made in the edge of a wide tension member under stress, it is found that the maximum stress is at the bottom of the notch and is slightly more than twice the average stress in the rest of the member. If a V notch is made the stress is usually greater and depends mainly on the radius at the bottom of the notch, and increases rapidly with increase of curvature: the inclination of the sides is also a factor in determining the intensity at the bottom of the notch.

If, for example, a very fine saw cut is made in the side of a member, which leaves the cut free from initial stress, a very small pull on the member causes stress beyond the elastic limit at the bottom of the cut, as is shown in a polariscope by the extreme concentration of colour bands at this place, and also generally by a dark patch round the apex of the cut due to breakdown of the material there: this latter effect persists when the load is removed. It is probable that scratches give similar effects, and the colour photographs which the speaker exhibited on the screen appeared to bear out his contention that, in minute indentations and notches of appreciable size, the effects of stress are so similar, where geometrical proportions are observed, that an effective study of scratches in materials can be made on notches of moderate size, provided care is taken to eliminate effects due to other boundaries: this can be ensured if the specimens are made large enough. In a ductile material the stress at a place of maximum intensity tends to spread over the neighbouring area, after the elastic limit is passed, and so equalise the stresses in that region. The character of the stress distribution therefore changes very rapidly after the yield point, but in a brittle material like glass the tendency appears to be towards maintaining the same type of distribution until fracture. It is therefore easy to cut glass with a diamond, since the effect of bending after a scratch is made is to cause a very high stress at the apex of the scratch, which does not tend to dissipate itself on the material in its neighbourhood, but is concentrated on the bottom of the notch as this latter is opened up by the bending. Nitro-cellulose cannot be cut in this manner unless in very thin sheets in which the depth of the scratch is an appreciable fraction of the thickness.

Mr H. Moore remarked that in the diamond cuts considered by the author the glass is not under any strain except that due to the cut. In Mr Preston's paper it was suggested that a cut produces a flaw into the top of which particles of glass are forced.

Mr H. S. Ryland said that it would no doubt be of very great interest to members of the Society if Prof. Cheshire would take an early opportunity of exhibiting by

means of polarised light the stresses produced by cutting glass and analogous substances during the cutting. The stresses produced during the cutting will probably be different from the settled stresses remaining after cutting.

Prof. Cheshire said he would be glad to do so.

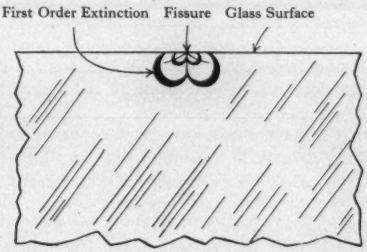
Mr F. W. Preston (communicated): The paper relates to a matter of so much interest and importance that I can only regret it is not of greater length.

The observations of the double refraction shown in the neighbourhood of the cut, when viewed lengthwise, are certainly accurate, but in some minor parts the conclusions drawn therefrom are open to exception.

The lines of principal stress shown in Fig. 3 are presumably approximately correct, but the only property of lines of principal stress is that the stress across such lines is wholly normal to them, and it is not correct to say that, because one line of principal stress represents a compression, therefore the other represents a tension. Both lines may represent tensions or both compressions, and the same line may represent compression in one part of its length and tension in another. It represents the direction of stress or a surface along which there is no shear, and the values of the stresses, in most cases, change continuously along the length and may change sign a number of times. This seems, however, to be recognised later in the text, where a violent horizontal tension is referred to, though the drawing seems to indicate it as a horizontal compression.

It follows, too, that since a surface along which there is no shear is a surface of principal stress, the free surface glass to air must be a surface of principal stress, and any lines of principal stress that intersect this surface must therefore cut it at right angles. The diagram given in the text shows lines of principal stress intersecting this surface at an angle of 20 to 30 degrees to the normal, and is therefore slightly inaccurate.

In circularly polarised light (which, in many respects, is the most convenient form in which to examine stress distribution, inasmuch as it eliminates the effect of the orientation of the nicol) the appearance is that shown in the accompanying sketch, which is so very much like the stresses shown under a bearing stress due to a point load that there is little doubt that the stresses are actually due to some small particles pressing into the surface of the glass in the neighbourhood of the cut.



Examined with the aid of a Babinet polariscope, the stresses appear to be of considerable complexity in the immediate neighbourhood of the cut, but at a very short distance from it the surface of the glass is in considerable compression, and a little below the surface at about the level of the bottom of the cut there can be detected the tension mentioned by Mr Twyman.

From these observations, and from the observations included in my paper, "The Structure of Abraded Glass Surfaces," I have little doubt that the whole of the stresses are caused by minute particles of glass wedging open the fissures made by the diamond.

Authors' reply (communicated): In reply to Mr Preston, we do not assume that the principal stresses at a point must necessarily be a compression in one direction and a tension in another direction. The double refraction of a stressed medium is indeed perhaps best referred not to stress, but to strain, since the variation of velocity of light through a transparent medium is a function of dilatation rather than of the accompanying stress. Thus all one can say from polarimetric observations is that in one direction there is maximum dilatation relatively to the direction at right angles. But it remains true that it is usually possible, as we think in the present case, by consideration to state whether or not the principal stresses at a point are compression in one direction and tension in the other. For instance, at the free surface there is no stress perpendicular to the surface, and the stresses at right angles to this must therefore be compressive. And if we follow these lines of relative dilatation inwards to the point where they approach closely to each other, we shall find (on the assumption that the lines crossing them at right angles are lines of compression—which we know to be true for the outermost ones) that a gradually increasing tensile stress arises along these lines. This must be so for the maintenance of equilibrium by the elements of volume represented by the quadrilateral spaces on the diagram.

But we do not gather that Mr Preston is in diagreement with us as to the character of the stresses present.

We have to thank Mr Preston for pointing out the slight error in the drawing of the diagram in the immediate neighbourhood of the free surface.

Examination by circularly polarised light has been used by us, and is of occasional convenience for a cursory examination, but it does not of course afford any knowledge of the direction of the stresses. Mr Preston's interpretation of these stresses, and of stresses in the neighbourhood of a ground surface, that they are occasioned by minute particles of glass wedging open the fissures, is a very natural one; and we do not ourselves regard our examination of the distributions of stress around a diamond cut as running counter to such an explanation; but we may frankly say that the explanation which has been borne in upon us by the observations we have made is quite different. Our own belief is that the stresses are occasioned by the removal of some of the glass by the diamond and the consequent disturbance of the forces of cohesion which before the cut had become equilibrated.

SOME MEASUREMENTS OF THE STRESSES PRODUCED AT THE SURFACES OF GLASS BY GRINDING WITH LOOSE ABRASIVES

By A. J. DALLADAY, A.Inst.P.
(Communicated by Mr F. Twyman.)

MS. received, 20th February, 1922. Read and discussed, 9th March, 1922.

The object of this work, which was carried out five years ago in the laboratories of Messrs Adam Hilger, Ltd., was to ascertain what was the actual value of the compression in the grey surfaces produced on glass by grinding with the abrasives in general use, viz. carborundum, corundum and Naxos emery, in various degrees of fineness. The grey was in each case produced by first grinding out with the abrasive under test any sign of the pits produced by the previous grade, and then finishing the surface with fresh abrasive, taking care not to work it down.

The specimen of glass used was Hard Crown, $\mu_D = 1{\cdot}518$, and was in the form of a flat strip of the following dimensions:

Length $l = 76$ mm.
Width $b = 11{\cdot}3$ mm.
Thickness $a = 3{\cdot}24$ to $3{\cdot}035$ mm. at various stages of grey.
Weight $w = 7{\cdot}0$ gm.

All faces were polished except the two small end faces.

The strip was supported at 45° with the vertical and observed along its width with vertically polarised light, crossed nicols and a sensitive one-wave mica plate. No sign of double refraction could be detected. One side of the strip was then ground with No. 80 carborundum, and again observed. The strip now appeared stressed as if it were being subjected to pure bending combined with extension; i.e. the neutral axis was straight, and nearer the polished than the grey surface; that part of the strip adjacent to the grey surface was in tension, the other being in compression.

It was assumed that this stress system was the result of a uniform compression in the grey surface itself, which, although intense, was so closely confined to the grey surface that it could not be directly observed by polarised light. The method adopted to measure this compression was to balance the stresses at a point in the glass by applying a known bending moment, and from this to calculate the value of the total thrust in the grey layer. The bending moment necessary to bring about this balance proved to be 125 gm. cm.

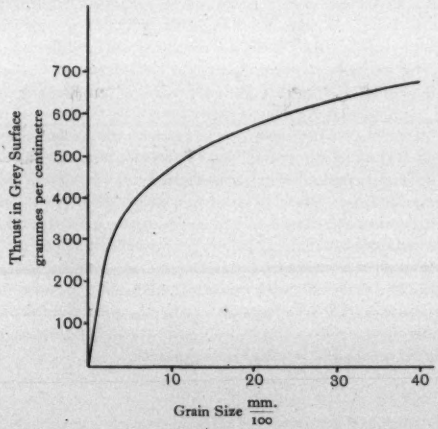
It can easily be shown that in a strip bent, as this one is, by a thrust in one surface, the neutral axis is shifted so that it is two-thirds of the distance from the stressed to the free surface; and from this it follows that if F is the thrust per unit

breadth in the stressed surface, and M is the bending moment produced, $F = 2M/ab$, a and b being the thickness and width of the strip, as before. This gives a value for the thrust of 680 gm. weight, or $6{\cdot}7 \times 10^5$ dynes, per unit breadth of surface.

No accurate measurements were made of the maximum double refraction present. These would have been difficult to make with any accuracy, and besides having no immediate bearing on the problem in hand, they can be deduced from the absolute value of the maximum stress present. The maximum double refraction close to the grey surface was probably about $\cdot 05\lambda$ retardation.

Having made the measurements on the coarse carborundum surface, the specimen was taken out and reground with No. 180 carborundum. The measurements were then repeated, giving a new value for the thrust in the grey surface. The results of these measurements are tabulated and plotted below.

Abrasive	Size of grain in $\frac{1}{100}$ mm.	Applied bending moment gm. cm.	Thickness (a) mm.	Thrust (F) in gm. per cm.
No. 80 carborundum	40	125	3·24	680
No. 180 carborundum	15	95	3·15	535
No. 200 corundum (natural) ...	10–12	84	3·11	480
Naxos emery (fine trueing) ...	5	66	3·06	380
Naxos emery (smoothing) ...	2·5	52	3·04	300
Naxos emery (fine smoothing)	1·5	36	3·03	210



It will be seen that as the coarseness of grain increases the thrust increases, at first in linear proportion, but very much more slowly as the coarser abrasives are used, so that although the coarse carborundum was nearly three times the grain size of the fine carborundum, the thrust it produced was less than 4/3 that

produced by the fine. It should also be noticed that there is apparently no discontinuity as a result of changing from carborundum to natural corundum and thence to ordinary Naxos emery.

The determinations of the balancing bending moments could be made with fair accuracy. The differences between successive determinations with the same grey did not exceed 5 per cent. in the coarse greys and 10 per cent. in the finest. It might at first be thought that the unsymmetrical position of the neutral axis would cause an error in estimating the balancing point but it will be seen that the errors produced above and below the neutral axis are in opposite sense, and therefore destroy one another. The results are the mean of three determinations and no greater accuracy is justified in view of the uncertain quality of the grey surface, which was ground by hand under ordinary workshop conditions.

As a check on the balancing method, the curvature produced by the bending moment was observed, in the case of the coarse carborundum grey, by placing the polished surface on an optical flat and observing the depression of the centre by means of the interference bands thus produced. It can be shown that a bending moment of 125 gm. cm. would produce on this strip a depression of 16·8 bands, assuming for Young's Modulus the value 6×10^{11} C.G.S. units. Experimentally the depression was found to be $15\frac{1}{2}$ bands.

The accuracy of observing these depressions is, of course, not great, particularly in the finer greys, with consequently fewer interference bands. The depressions are interesting, nevertheless, as showing directly the effect of a grey surface on the perfection of figure of the opposite surface of optical work, which is a factor liable, on occasion, to be overlooked. As an illustration of this may be mentioned the case of a galvanometer mirror, one inch diameter, polished on one side to a radius of curvature of one metre, and subsequently reduced in thickness to about half a millimetre at the centre. This surface was finished with a fine grey, and when the radius was checked it was found to be 85 centimetres. Repolishing the back restored it to its correct radius of 100 centimetres.

In conclusion I wish to thank Messrs Adam Hilger, Ltd., by whose permission I am enabled to present these data, and at the same time I must apologise for the incompleteness of the subject matter. I am fully aware that there are many criticisms possible, for although the results, so far as they go, are accurate, the object of the experiments did not call for accurately standardised conditions, nor did they justify a complete investigation of the effects of varying those conditions. I present the paper in the hope that it may prove of interest in the light of Mr F. W. Preston's paper on "The Structure of Abraded Glass Surfaces."

DISCUSSION

Mr F. W. Preston (communicated): The paper seems so complete and to the point that I do not think I can add much to it immediately. The quantitative measurement of the quantity F, which has the dimensions of surface tension, and may be called the surface compression, marks a distinct advance. The proof that

F is a fairly definite function of the grain size of the abrasive is interesting, but it seems rather remarkable that it should be independent of the nature and other properties of the abrasive. For example, one would expect it to vary somewhat with the angularity of the abrasive. It would seem that the quality of an abrasive might almost be defined in terms of its F value.

I have not hitherto been sure that it was valid to calculate F as $2M/ab$; it assumes that the stress at a point in the body of the glass varies as a linear function of the distance of the point from the grey surface. The theoretical justification for such an assumption seems to me complete, but hitherto I have not been able to make it square with the experimental evidence. This straight line law would require as a corollary that if a parallel piece of glass be greyed equally on both faces then the body of the glass is in uniform tension, the tension being equal at all points throughout the thickness. My impression has been hitherto that the double refraction was slightly greater near the grey surfaces than at a distance from them.

In view of this paper I have now repeated my observations, and am satisfied that Mr Dalladay is right and that the straight line law is valid. In particular I believe that a piece of glass greyed on both faces is in uniform tension and the strain is not localised near the faces. Our previous observations that the double refraction sometimes seems somewhat greater nearer the faces must, I think, be attributed to secondary causes, such as internal total reflection which depolarises the light a little. Mr Dalladay's view must, therefore, be accepted as a correction to the non-linear relation more or less implied in my paper.

I am not clear just how M was determined in Mr Dalladay's experiments, for the application of a neutralising bending moment will not neutralise the general extension as well as the bending moment with which it is combined. The ideal way would appear to be to grey both faces and then to apply a longitudinal compression of $2Fb$ to the specimen as a whole. This should not present any great difficulty as F is of the order of 4 lbs. per inch of width. When the correct thrust was applied the whole field would become uniformly dark again between crossed nicols.

It remains to explain the shape of the curve obtained when the surface compression is plotted against coarseness of abrasive. Let the thickness of the grey layer be t, and let the ultimate strength of the glass be f. Then, by reason of the mode of formation of the grey layer, it follows that the stress there must approximate to f*. Then if f is constant throughout the grey layer, $F = ft$.

Further, if the principle of similarity were valid in such cases, t would be proportional to the grain size of the abrasive. Thus $F = kfD$, where D is the diameter of the abrasive grain. Thus if f were a constant independent of t, the compression F would be proportional to the grain size. It is, however, well known that very small particles of matter, very fine wires, and so on, have a strength much greater than the principle of similarity would suggest; this may account for the departure from a straight line law for this graph.

On the other hand, the degree of accuracy claimed for the measurements of M, and the inevitable errors involved in the determination of grain size D, seem to

* See also Twyman, *Proc. Opt. Conv.* (1905), p. 52.

be such as do not definitely prove that at first the thrust increases in linear proportion with grain size. A law of the form $F = 210D^{\cdot 33}$ might almost equally well fit the case.

Mr T. Smith asked whether the value of Young's Modulus used was that for bulk or for surface layers, and whether the effects due to the greyed rim of the lenses extended sufficiently far to affect, for example, the aberrations of a microscope objective.

Mr H. S. Ryland said that in proof plane work colour changes sometimes took place which could not be explained by temperature variations. The example of the galvanometer mirror mentioned in the paper probably afforded a satisfactory explanation.

Mr F. Twyman (partly communicated): The value adopted for Young's Modulus was taken from published tables, and was not determined for the actual sample used.

I have never been able to find that the resolving power of an echelon grating is affected by the presence of greyed edges. The double refraction only amounts to a small fraction of a wave length, and I do not think, therefore, that the definition of a microscope objective would be affected by the greyed edges.

Mr Preston has kindly associated my name with the effect of strain observed in glass which has ground surfaces. Mr Preston has also pointed out to me in correspondence that I did not either very clearly, or, as I think now, even correctly describe its details.

I wish, therefore, now to state that I am in entire agreement with Mr Dalladay as to the nature of the distribution of stress in the cases considered, and indeed have been in entire accord with him on this subject throughout the course of the experiments, with the progress of which I was continuously in touch.

Author's reply (communicated): Mr Preston has objected that the general extension would cause an error in determining the balancing point. The point of balance chosen was on the neutral axis and, as I have pointed out in the paper, the errors produced above and below the axis are of opposite sense, and therefore cause no bias in either direction.

The ideal way would certainly appear to be to apply a uniform longitudinal compression to a strip greyed on both faces. It has, however, been found by others to be an exceedingly difficult matter to apply an absolutely even compression of this kind; so much so, that the method has been abandoned whenever possible. The expenditure of time and the fitting up of the necessary apparatus were not justified by the objects of the experiments, hence the simple method adopted.

With regard to the errors involved in measuring the grain size, the abrasives used were rendered very uniform in size by elutriation, and their mean diameters measured microscopically. The errors resulting from this cause, are, therefore, small.

THE BARR AND STROUD 100 FT. SELF-CONTAINED BASE RANGEFINDER

By JAMES WEIR FRENCH, D.Sc.

Read and discussed, 12th January, 1922.

SUMMARY. Depression, two-position, and more recently self-contained types of rangefinder have been employed for coast defence purposes. The first two types are subject to several disadvantages which have led to the introduction of the 100 ft. base self-contained instrument. The rangefinder has a new type of triple field. It is carried upon a mounting comprising two trucks running upon a roller path of 50 ft. diameter, the trucks being connected by a rigid horizontal framework. Upon the trucks are carried cantilevers from the ends of which are suspended cradles having special bearings within which the rangefinder rests. Training is done by power or by hand. During extensive tests the uncertainty of observation at a range of 31,000 metres did not exceed 20 metres.

For Coast Defence purposes, the rangefinders employed have been of the depression and two-position, and more recently the self-contained base types. In the case of the depression rangefinder the plane of triangulation is vertical, the base of the triangle being the height of the observation telescope above a datum plane. It is an instrument that is particularly suited for high positions, as the accuracy depends upon the length of the vertical base. Observations are made upon the waterline of the ship, lying in the datum plane which varies with the tide. Means for making the necessary tide corrections are generally embodied in the instrument. As the plane of triangulation is vertical, the rays of light pass obliquely through the various strata of air and are subject to a considerable amount of refraction which can only be approximately estimated.

Rangefinders of the two-position and self-contained base types have their bases approximately horizontal. They can operate upon any visible part of a ship, such as the mast, and can be installed at any height. As the beams of light to the two ends of the base pass through the same strata of air, the refraction errors are differential and usually inappreciable.

For the long range firing that is now essential, depression rangefinders are inadequate. In long range firing the elevation of the gun may approach 45° and the shell necessarily falls at a much greater angle than is the case when the range is shorter. To hit the target, therefore, the longer the range, the less is the allowable range error. On the other hand it follows from the principle of rangefinding from a fixed base length that, other things being equal, the unavoidable uncertainty of range determination varies directly as the square of the distance. For long distance

work, therefore, it is necessary to determine the range with very particular accuracy. As the accuracy of range determination is proportional to the base length of the instrument, it follows that the base length must be increased in a greater proportion than the maximum range of firing to be controlled.

For long range firing the two-position rangefinder, having a base length, it may be, of a mile or more, has been extensively employed. But self-contained base rangefinders will most probably supplant the older two-position type, which suffers from several serious disadvantages.

With a self-contained rangefinder it is possible to take ranges rapidly and at very frequent intervals. In gunnery this is essential. Accuracy and rapidity of fire depend upon a continuous stream of target ranges and bearings being communicated to the Fire Control Station, whence a continuous stream of tangent elevations and gun bearings can be communicated to the individual guns of the various batteries.

Being self-contained, there is less chance of the rangefinder being directed upon a wrong target, as is the case in two-position installations where the targets may present entirely different aspects to the two stations and in the latter installations there is the additional danger of the instruments at the two independent ends of a long base being directed upon different parts of the target.

Further, the use of a self-contained rangefinder avoids the necessity for having telephonic or other communication between two stations widely separated for the purpose of identifying the target and for the transmission of data.

An enemy will probably approach its objective on a bearing least suited to the defenders and at the highest possible speed. It is essential, therefore, that the Coast Defence rangetakers should be able to transmit ranges as quickly as possible after the enemy becomes visible. When the rangetaking depends upon two widely separated operators who must communicate telephonically with each other in order to identify the particular target upon which both must operate, delay is inevitable. The self-contained rangefinder has the great advantage that the operator is independent. His instrument may indeed be directed upon the particular target by the commanding officer, the only duty of the rangetaker being then to keep the images in coincidence, and automatically transmit the ranges obtained.

There is a much wider choice of position for the installation of a self-contained rangefinder, which can be placed with an all-round view at any convenient distance from the battery. In the case of a two-position installation it is not always possible to find convenient sites that will provide the desired length of base disposed at right angles to the direction from which attacking ships are most likely to approach, and generally the area that can be effectively kept under observation is comparatively restricted. It will be readily understood that the base and therefore the accuracy of observation in the case of the self-contained rangefinder are independent of the direction of the target, whereas in the two-position installations the maximum effect of the base is obtained only in the region of one bearing.

It might be thought that a rangefinder of 100 ft. base could not approach the accuracy of a rangefinder having a base length of, say, one mile. It should be

remembered, however, that in the self-contained rangefinder the two images that are to be brought into alignment or combined are brought closely together in the field of a telescope instead of being independently viewed by two operators. In the former case the uncertainty of observation is of the order of $\frac{1}{2}$ second of angle when the magnifying power is 30 diameters, whereas in the latter case the uncertainty of observation is unlikely to be less than 30 seconds. The one observation is, therefore, about sixty times as accurate as the other and on this account alone the 100 ft. self-contained instrument should be able to compete with a two-position instrument having a base of one mile.

But there are other conditions that adversely affect the accuracy of the two-position instrument, such as the difficulty of operating with the necessary accuracy upon the same part of the target and the greater atmospheric effect on two beams of light separated by a mile, as compared with two beams separated by 100 ft.

There is, further, the question of cost. The self-contained instrument involves the acquisition of one piece of ground and the maintenance of one establishment,

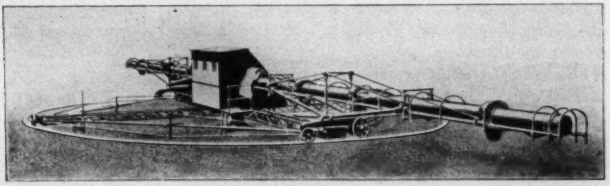


Fig. 1

whereas the two-position instrument involves the purchase of two or more separated pieces of ground, the maintenance of two establishments, and an expensive installation of electrical interconnections.

The reason for the introduction of the 100 ft. rangefinder, which I shall now briefly describe, will be evident from the foregoing remarks.

The rangefinder as at present installed is indicated in Fig. 1. It comprises a rangefinder proper of 100 ft. base length supported upon cradles and cantilevers carried by two trucks connected together by a rigid horizontal framework and running upon a circular track, the diameter of which is 50 ft. Over the centre part of the rangefinder there is provided a cabinet for the weather protection of the three operators. The man beside the cabinet using the small rangefinder of 80 cm. base is introduced to give an indication of relative size.

Although the title of this paper refers only to the rangefinder, it is the mounting that is of most interest to a designer, and indeed the design and construction of large instruments involve more engineering problems than optical. The general principles of the rangefinders are already well known and it is only necessary to refer to the special features.

The base length is 100 ft. and the tube diameter 14 ins. It comprises five sections, the adjacent ends of which are provided with coupling pieces and keyways. Previous experience with such couplings demonstrated that the sections of a rangefinder after adjustment could be taken asunder and re-assembled without necessitating workshop readjustment.

This particular instrument works upon the coincidence principle of observation, the field of view being as indicated in Fig. 2, in which the two small fields above and below the separating line contain images formed by light proceeding from the ends of the rangefinder respectively, while the surrounding field is formed by a third telescopic system, the aperture of which is situated at the centre of the rangefinder. The main field has an angular width of 1° 30′ and, as the highest magnification is 28 diameters, the maximum apparent field is 42°. The rangefinding fields have an angular width of 17 minutes.

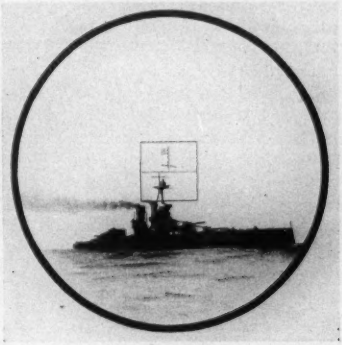


Fig. 2

To provide a large central field would have necessitated the use of very large end reflectors and, if the intention had been to produce a telescope with a large field, large end reflectors would have been constructed; but the function of the instrument is to measure ranges as accurately as possible and large optical parts in instruments that have to make very precise measurements are not desirable. Therefore comparatively small end reflectors and a small field have been provided in combination with means for so training the whole instrument that the object can be easily kept within the limits of the rangefinder field.

For the deviation of the beam of light upon which the measurement of the range depends, there are provided two prisms set transversely to the beam and capable of rotation in opposite directions about the longitudinal axis of the beam. These prisms are mounted near one of the end reflectors. If a translating prism near the centre of the instrument had been employed, as in other standard Barr and Stroud rangefinders, it would have been necessary to increase the size of the end reflector concerned to an undesirable extent and to have displaced it. For the operation of these prisms from the centre of the instrument there is provided a shaft specially designed to avoid any lost motion, since such lost motion would affect the scale readings, the scale gear and the prisms being situated at opposite ends of the shaft. To simplify this particular problem, the transmission shaft is caused to make a large number of revolutions between the extreme limits of the scale.

Sometimes the opinion is expressed that the thermal extension of the base must seriously affect the range determination. If θ is the parallax angle, B the base and R the range, then $\theta = \frac{B}{R}$ and hence $\frac{dR}{R} = \frac{dB}{B}$, that is, the range and base vary by the same percentage amount. The percentage increase of the base is very small and its effect on the range determination is negligible.

If simple end reflectors were employed, an extremely small temperature difference between the front and rear sides of the tube would be disastrous. This difficulty is overcome in practically all rangefinders by the use of double or so-called pentagonal reflectors, the principle of which is well known. Bending of the tube in the plane of triangulation has then no effect upon the horizontal direction of the rays, provided the double reflectors are correctly adjusted. But differential bending in the vertical plane results in a vertical separation and also a very slight rotation of the images in the field of view. When the shop adjustments are correctly made, the vertical separation of the images due to a vertical differential deflection of the tube of several minutes is not accompanied by any appreciable displacement of the images in the direction of the separating line which would represent an error of range determination. But as the deflection increases, the effect upon coincidence becomes increasingly apparent.

Correct support of the instrument in the vertical plane is of considerable importance as, notwithstanding the sag of the tube within the supports, the minimum planes of the end reflectors should lie parallel to the optical axis of the central telescope. To realise this condition the comparatively flexible tubular rangefinder is made to rest within eight supports, the separations of which are adjusted by calculation to suit the weights and stiffnesses of the various rangefinder sections and suitable positions for the cradle supports are obtained by modifying the positions of the heavy couplings, thus altering the distribution of the weight.

A certain amount of elevational movement, that is, about the longitudinal axis of the rangefinder, is required to enable the instrument to be directed upon targets at various elevations and in this particular instrument a maximum depression of 10° and a maximum elevation of 70° are provided, so that aerial craft may be observed upon.

Considering that the instrument is supported within eight bearings, it was a matter of some concern as to what elevating force would be required. If the force required were great, there would be danger of twisting the tube and thus introducing errors. Very particular attention was accordingly paid to the design of the bearings and the force required to elevate is astonishingly small; it is not more than about 4 ins. lbs.

When the elevating handwheel is removed the rangefinder can be elevated by grasping and turning the shaft, which has a diameter of about 1″. This feature is mentioned as indicating what can be done by careful design in the elimination of frictional resistance.

The eight supports are mounted on the ends of four cradles, two of which are suspended from the ends of the right hand cantilever and the other two from the

left cantilever. The cradles swing freely in the vertical plane but are constrained in the horizontal plane. It was not found necessary to provide any self-aligning devices in this plane. But means were provided for aligning the supports in the first instance, as will be described later.

One of the bearings is indicated in Fig. 3. Within the bottom case there is mounted a roller carried upon ball bearings, all the parts in question running in oil. The whole load is carried by this wheel. Within the upper case there are provided two guidance rollers which can be set by means of adjusting screws, when the top cover is removed. Special packing rings are provided on either side of the bearing to prevent the entrance of moisture. No alteration of the resistance has been observed during the last eighteen months, notwithstanding that the instrument has been exposed to weather of all kinds.

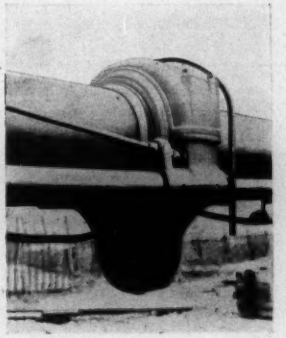


Fig. 3

Differential twisting of the rangefinder tube in the process of elevation must be reduced to a minimum. It was decided, therefore, to apply the forces of elevation at two positions on either side of the centre and so to adjust the positions that the twist of the two ends relatively to the central section would be the same. The arrangement is indicated in Fig. 4. Racks concentric with the rangefinder axis are mounted upon the bearing case. The elevating handwheel is attached to the rangefinder. The force applied to the handwheel is communicated through coupling shafts to pinion wheels which gear with the racks. Means were provided originally for rotating one of the coupling shafts more than the other if in the field of view there should appear any vertical displacement of the images, but the calculated positions were found to be so nearly correct that these special means were not required.

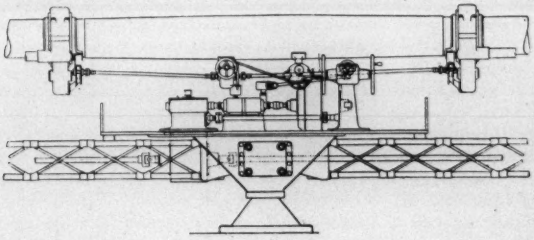


Fig. 4

This figure, but more particularly Fig. 5, shows also the training gear which may be briefly described. The electric motor shown in the plan drives a Williams Janny variable speed oil transmission gear which is connected to a special gear box. From the lower end of this gear box there proceed the two driving shafts which communicate the motion to the driving wheels of the two trucks. The trainer is seated at the training pedestal. By means of the small handwheel at the front he controls the valve of the Williams Janny gear. Thus, while the motor is running continuously, if the index on the top of the pedestal is in its mid zero position, the swash plate of the Janny gear is normal to the axis and no rotational motion is transmitted to the gear box. If the pointer is displaced to one side or the other, the swash plate is suitably inclined and the motion is communicated through the gear box to the driving wheels at the appropriate speed

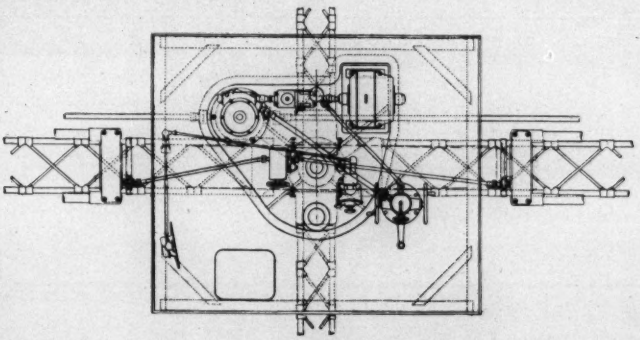


Fig. 5

in one direction or the other. The maximum slewing speed is 180° per minute. The minimum slewing speed that has been definitely recorded is $\frac{1}{2}$ minute of angle per minute of time.

By means of the two handwheels at the side of the pedestal, the trainer can operate the mounting at a maximum speed of about 20° per minute quite independently of the power training and while the latter is in action. If the trainer finds that the rangefinder is gaining on the target, he may slow down the motion by hand training in the reverse direction until such time as he thinks it desirable to reset the power gear. The hand gear may of course be used in the event of failure of the current supply.

Longitudinally the position of the rangefinder is controlled by the abutment of two sector flanges attached to the tube against ball bearing wheels carried on the mounting, as indicated in Fig. 6. One of the two cantilevers, from the ends of which are suspended the cradles carrying the bearings within which the rangefinder

rests, is indicated in Fig. 7. The cantilever is pivoted to a two-wheeled truck which runs upon the roller path, the driving force being transmitted to the larger of the two truck wheels through the intermediary of a friction plate clutch, the

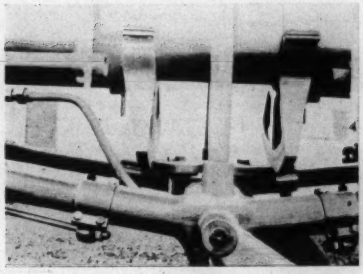


Fig. 6

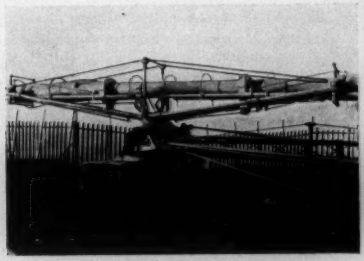


Fig. 7

purpose of which is to make the training movement more uniform. About three-quarters of the load is carried by the driving wheel. Fig. 7*a* is another view of the right-hand cantilever which shows more clearly the cradle arrangement and also the various tension members. It also shows the large driving wheel of the truck and the smaller tail wheel.

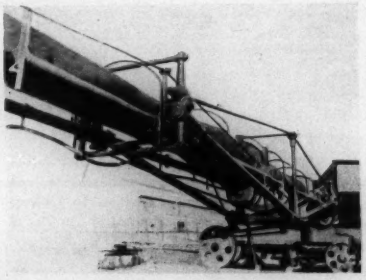


Fig. 7*a*

Probably the most important feature of the whole mounting is the horizontal framework associated with the two trucks which carry the cantilevers, cradles, and the rangefinder. From the plan, Fig. 8, it will be seen that the frame is quadrilateral. It comprises a rigid lattice-work girder set at right angles to the rangefinder and bolted securely to the central pivot sleeve, together with two radial members

rigidly fixed at their outer ends to the two trucks and pivoted at their inner ends to the central pivot sleeve. Tension members connect the outer ends of these cross and radial girders and serve to place the whole framework in compression, there being provided suitable tension screws and nuts, all of which are finally enclosed to protect them from the weather. It should be observed that the central pivot carries only the weight of the cabinet together with the operators and a portion of the weight of the radial members. It does not carry any of the weight of the rangefinder and other parts, which is concentrated over the roller path. The rigid framework cannot readily be distorted by the training forces applied to the trucks which, therefore, do not tend to distort the rangefinder. By means of the tension

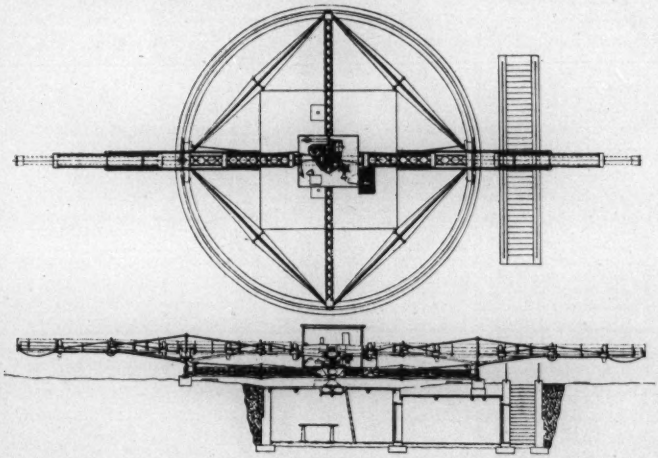


Fig. 8

members the pivoted girders associated with the trucks can be adjusted in the first instance so as to bring the rangefinder bearings into alignment. It will be understood, however, that, so far as such adjustments are concerned, reliance was placed more upon accuracy of workmanship than upon the provision of adjustments.

This illustration shows the underground chamber which will contain the Fire Control Apparatus but does not indicate the special tackle provided for the erection of the instrument. To facilitate transport to the site, the weights of individual parts were limited to one ton.

Fig. 9 is a partial internal view of the cabinet. Accommodation for three operators is provided. The rangetaker controls the elevation and maintains the

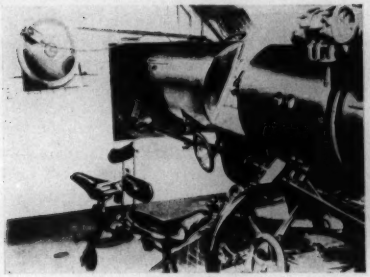
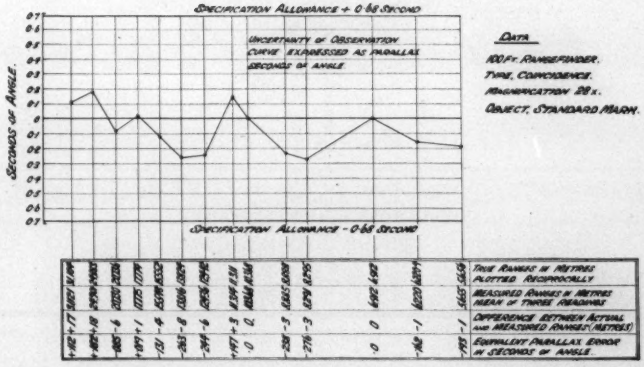


Fig. 9


SPECIFICATION ALLOWANCE + 0·68 SECOND
UNCERTAINTY OF OBSERVATION CURVE EXPRESSED AS PARALLAX SECONDS OF ANGLE
SECONDS OF ANGLE.
0·7 0·6 0·5 0·4 0·3 0·2 0·1 0 0·1 0·2 0·3 0·4 0·5 0·6 0·7
SPECIFICATION ALLOWANCE - 0·68 SECOND
DATA
100 Ft. RANGEFINDER.
TYPE, COINCIDENCE.
MAGNIFICATION 28 x.
OBJECT, STANDARD MARK.
TRUE RANGES IN METRES PLOTTED RECIPROCALLY
MEASURED RANGES IN METRES MEAN OF THREE READINGS
DIFFERENCE BETWEEN ACTUAL AND MEASURED RANGES (METRES)
EQUIVALENT PARALLAX ERROR IN SECONDS OF ANGLE.

Results of Test of 100 Ft. Base Rangefinder

Manufactured by

Barr & Stroud Ltd

Glasgow

Fig. 10

images in coincidence in the field of view. There is provided a uniform scale gear which converts the particular scale of the instrument, which is a combination of reciprocal and sine curves, to a scale of uniform divisions which are indicated upon a follow-the-pointer dial on the side of the cabinet, controlled by a separate operator who automatically transmits the ranges electrically to the Fire Control station. The third operator is seated at the training pedestal previously described, and is provided with a special sighting telescope, shown in position above the rangefinder. Bearings are transmitted directly from the rangefinder to the Fire Control station.

At the rear of the cabinet there is installed a desiccating chamber and air circulation fan. Whenever the electrical motor is running, dry air is being continuously circulated through the rangefinder. It is found that when any fogging is present, owing to the instrument having been out of use for some time and exposed to damp weather, the desiccating arrangements clear the fields within about a quarter of an hour from the time the current is switched on.

Finally I should like to say a few words regarding the accuracy attained during the official trials, the particulars of which are indicated in Fig. 10. It should be remarked that the readings in question were taken during two consecutive days. Each point marked is the mean of three readings; every reading taken was embodied in the chart.

In estimating the probable uncertainty of observation, it was thought that an allowance must be made for the long column of air within the rangefinder tube, and therefore a specification allowance of $+0{\cdot}68$ second was adopted. These values are indicated by the top and bottom lines in the diagram. Ordinates represent seconds of parallax angle in steps of $\frac{1}{10}$ second; abscissae represent ranges, as indicated below the diagram, the lowest range, at the right hand side, being 5656 metres, and the highest range 31,104. The magnification employed was 28 diameters; actually the instrument is provided with two additional powers of 20 and 16. The angles resolved by the eye are, therefore, 28 times those indicated on the chart. It will be seen that in no case did the error exceed $0{\cdot}27$ second and, therefore, in no case did the observer's eye fail to resolve an angle of at the most $7{\cdot}6$ seconds, notwithstanding exceptionally bad observing conditions, due to the presence of smoke and hot air between the object and the rangefinder. It will be seen that at 31,000 metres the uncertainty of observation was 17 metres and at the lowest range about 1 metre. If the average equivalent parallax error is taken, it will be seen that the average resolving power of the unaided eye, taking sets of three readings at each measurement, was 4 seconds of angle.

These particular official tests were made by operators who had only one day's experience of the instrument and who acted quite independently of the makers' staff. The results obtained by ourselves during a considerable number of months in all kinds of weather have been very comparable with those illustrated and often considerably better.

The optical system of the rangefinder is shown in Fig. 11.

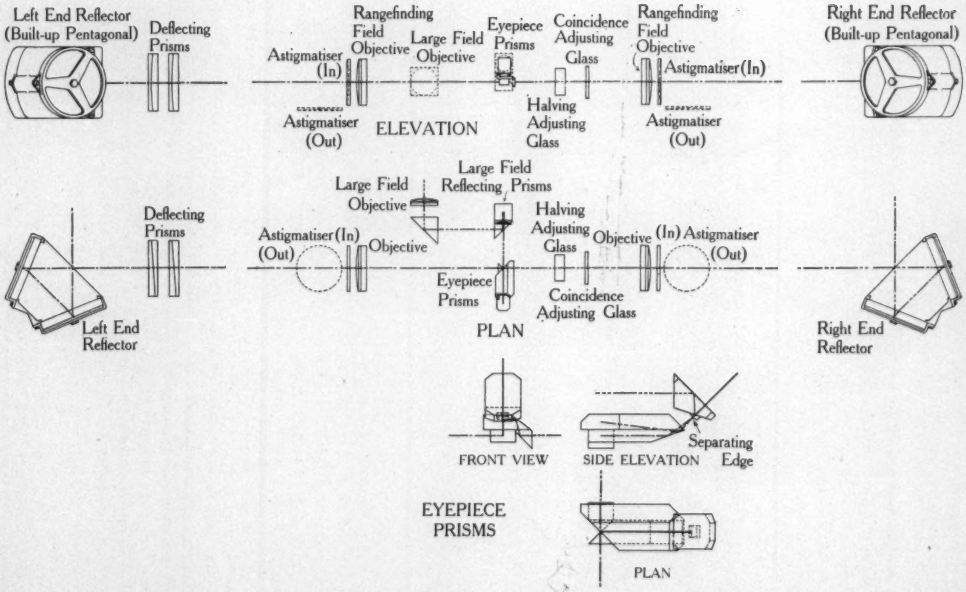

Left End Reflector (Built-up Pentagonal)
Deflecting Prisms
Rangefinding Field Objective
Astigmatiser (In)
Large Field Objective
Eyepiece Prisms
Coincidence Adjusting Glass
Rangefinding Field Objective
Astigmatiser (In)
Right End Reflector (Built-up Pentagonal)
Astigmatiser (Out)
ELEVATION
Halving Adjusting Glass
Astigmatiser (Out)
Deflecting Prisms
Large Field Reflecting Prisms
Large Field Objective
Halving Adjusting Glass
Astigmatiser (In) (Out)
Objective
Objective
(In) Astigmatiser (Out)
Eyepiece Prisms
Coincidence Adjusting Glass
PLAN
Left End Reflector
Right End Reflector
FRONT VIEW
SIDE ELEVATION
Separating Edge
EYEPIECE PRISMS
PLAN

Fig. 11

DISCUSSION

Lt.-Col. Gifford enquired as to the effects of temperature on the accuracy of the instrument.

Instructor-Commander T. Y. Baker asked if any radical alterations had been made in the optical system and if large pentagonal prisms or built up systems of mirrors were employed.

Professor Cheshire suggested that the author might insert in the paper a simple diagram of the optical system.

The President (Mr R. S. Whipple) said that apart from astronomical instruments this was probably the largest scientific instrument yet constructed.

The author, in reply, said that the instrument is designed on the unit system. Each part is tested for temperature effects, charts being plotted and corrections made where necessary. When the errors have been reduced to a minimum the instrument is assembled and again tested. It is possible to diagnose the temperature charts by making tests of different kinds. The temperature trouble is bound always to be a serious one.

In large rangefinders one is forced to use end reflectors of the built up type, owing to conductivity difficulties and to the difficulty of obtaining large blocks of glass of sufficient homogeneity. In the present instrument the reflectors rest against the faces of gun-metal castings. Invar, silver, and other alloys had been tried, but it was found sufficient to use a material which would allow heat to get as quickly as possible into the proper places.

No great alterations had been made in the optical system. Rotating prisms had been introduced and the eyepiece prisms were somewhat different owing to the introduction of the triple field of view.

He agreed to supply a diagram of the optical system, which is now added to the paper and is sufficiently self descriptive.

One point of interest to engineers was that in the design of the instrument every part of the mounting was drawn full size. This facilitates the correct visualisation of the strengths of the various parts. A half scale drawing often proves to be very deceptive in this respect. He believed that even in the design of the Forth Bridge full scale drawings of all main details had been made.

A CRITICISM OF THE NODAL SLIDE AS AN AID IN TESTING PHOTOGRAPHIC LENSES.

By T. SMITH, M.A., F.Inst.P.
and JOHN S. ANDERSON, D.Sc., F.Inst.P.
(Optics Department, National Physical Laboratory.)

MS. received, 19th December, 1921. Read and discussed, 9th March, 1922.

SUMMARY. The nodal slide is only convenient for the examination of lenses over their entire field when these are of normal type.

Collimators and lenses should in general be directed to pass through all useful light as symmetrically about their axes as possible. Failure to secure this condition may lead to an underestimate of the excellence of good lenses as well as an underestimate of the gravity of the faults present in bad lenses.

The nodal slide may with advantage be supplemented by suitable linkages, which enable all the special conveniences of the nodal slide to be retained. Some of the possible modifications are discussed.

The nodal slide is now very well known as a device of considerable utility both in University Optical Laboratories for experiments on the fundamental properties of lenses, and in works and testing laboratories for the detailed examination of lens systems for the determination of their aberrations. For a very extensive range of systems no difficulties occur in its use, but in any system in which there is a considerable axial displacement between the entrance and exit pupils on the one hand and the nodal points on the other the defects of the nodal slide become very apparent, and frequently apparatus into which it is built as an integral part may be found useless for conducting a test. These difficulties have now been appreciated for several years at the National Physical Laboratory, but the fact that at least two expensive testing outfits have been built in this country within the last year or two without any modification in the design which avoids the defects of early patterns suggests that a note calling attention to the limitation of this mechanism may not prove without value.

In recent years notable improvements have been effected in the design of fixed focus telephoto lenses, and the best of the new designs will bear comparison both for speed and for freedom from aberrations with the best known anastigmats of normal construction. There is thus at the present time special interest in the examination of these lenses, and some observations on a sample recently examined will serve to illustrate the failure of the nodal slide. The lens examined was of focal length 37·1 cm., the principal foci were distant from the vertices of the first and last surfaces 48·6 cm. and 18·7 cm. respectively, and the clear apertures of these extreme surfaces were 6·4 cm. and 4·7 cm. respectively. The lens was intended to cover a field of diameter 20·4 cm., corresponding to an angular field of about

30·8°. When the lens was correctly adjusted with its back nodal point on the axis of rotation of the slide, the observations made in the examining microscope showed that the illumination of the image fell to zero 13° off the axis corresponding to about $\frac{1}{3}$ of the field that was required to be covered with satisfactory intensity. Actually the lens covered the specified plate with ease, and the incorrect impression derived from the observation was caused through the use of the nodal slide under conditions for which it is unsuited. The actual state of affairs can be readily appreciated on reference to the accompanying figures.

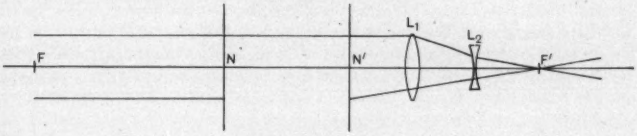


Fig. 1

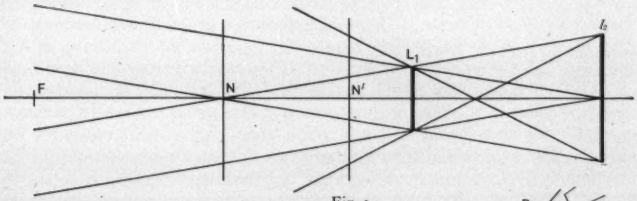


Fig. 2

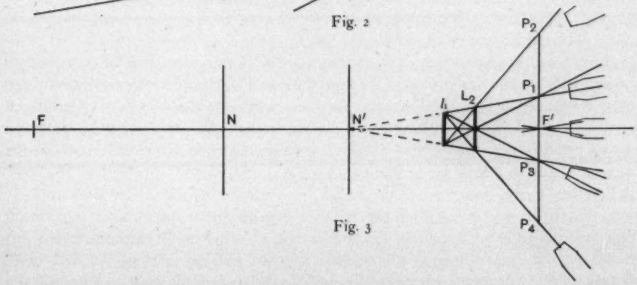


Fig. 3

Fig. 1 illustrates diagrammatically a telephoto lens composed of two lenses of focal lengths $f/2$ and $-f/3$ with their inner principal planes separated by the amount $f/3$. The four Gaussian constants of the system are consequently

$$1/f,\ 2,\ \tfrac{1}{3},\ -f/3,$$

satisfying the identity $\quad 2 \times \frac{1}{3} - (1/f)\,(-f/3) = 1.$

The meaning of these four quantities is that the focal length of the combination is f; the distance of the first principal focus F from the first lens L_1, or more exactly from its external principal plane, is $2f$; the distance L_2F' from the last lens L_2 to the second principal focus F' is similarly $f/3$; and the separation L_1L_2 has the assumed value $f/3$. It follows that the first nodal point N lies a distance f in front of L_1, and the second nodal point N' a distance $2f/3$ in front of L_2. In other words if the lens flange is placed in the centre of the mount, the focal length will be twice the camera extension. A marginal ray incident parallel to the axis is drawn in the upper part of Fig. 1 in its actual path to the focus F', and in the lower part the corresponding Gaussian path for the complementary ray is traced.

This figure gives, when aberrations are neglected, all that is required to show how an incident ray, if it traverses the system at all, will be refracted. To find which rays are enabled to get through, the windows to which the lenses are equivalent must be constructed. It may be assumed that at full aperture, the condition under which the lens has to be examined, the only effective stops are the two lens cells where they are met by the extreme surfaces. Neglecting the curvature of these surfaces and assuming that Gaussian laws of image formation need alone be considered, the last three of the Gaussian constants give the positions and sizes of the equivalent stops. For the object space the front lens rim itself serves as one stop, and l_2, the image of L_2, as the other. The distance of the latter from L_1 is $(-f/3)/\frac{1}{3} = -f$, the minus sign indicating that l_2 lies on the side of L_1 away from the object space. The image is three times the size of the lens itself and is not inverted. The equivalent windows as far as the object space is concerned are shown in Fig. 2. Similarly for the image space one stop is the actual rim of L_2, and the other is l_1, the image of the rim of L_1, which is distant $(-f/3)/2 = -f/6$ from L_2 (and thus on the side remote from the image space), and is half the size of L_1. The equivalent windows for the image space are drawn in Fig. 3.

It will not be unreasonable to suppose that each component lens has an aperture numerically equal to one third of its focal length. The complete lens will then have an aperture of $f/6$. In the diagrams distances normal to the axis have been drawn on a scale twice as great as that for distances along the axis, in order to avoid undue crowding of the various lines it is desired to show. The difference in the two scales has of course no influence on general deductions to be drawn from the diagrams. On the assumption just made the diameter of l_2 is twice that of L_1. If then the object is at a considerable distance it is seen that full pencils of rays can only be utilised over the angle limited by the rays extended to the extreme left of Fig. 2. Beyond this angle the second aperture eclipses part of the first, with the result that the light intensity falls off rapidly and diminishes to zero for the angle limited by the short rays. Evidently if a collimator were used with an artificial star as a source and the two systems were so connected that the collimator axis passed through N, the useful light would pass through the collimator objective symmetrically when the two axes coincided, but on any change of direction want of symmetry would be introduced, and at the extreme angle for full illumination in the case illustrated the useful light would pass through one half only of the

collimator objective. As the rotation continued further the light that could be utilised would proceed to the margin of the collimator aperture. Unless the aperture of the collimator objective were at least twice the diameter of that of the lens under test, the latter would not be filled with light even for the central portion of the field, and in the outer field no light at all would reach the lens. Even if the aperture of the collimator suffices to avoid all these difficulties, the use of very oblique partial pencils for testing the outer portions of the field which the supposed constraint to its axis implies offers ample ground for rejecting this arrangement. In all auxiliary apparatus oblique passage of a beam tends to produce increased errors or uncertainty. The only satisfactory course lies in directing the collimator axis to a point much nearer to L_1 than N. The vertex of L_1 would be the most satisfactory point if the field involved is only that for full pencils. If the field is larger than this a point rather more removed from N is desirable, but with larger fields the point selected is a compromise between the vertex of L_1, the ideal point for full pencils and the gradually receding points preferable for eclipsed pencils. The limits to the range are the vertex of L_1 and the point where the axis is met by crossed lines from the extremities of the two apertures, which is the ideal point when the one aperture is all but eclipsed by the other.

Exactly similar considerations apply in the image space. The pencils involved arise from the aperture l_1 and are bounded by cones whose vertices lie on the plane through F' normal to the axis. The various lines in Fig. 3 for this space correspond to those of Fig. 2 for the object space. Thus the extreme pencils of full illumination have an edge bounded by lines through the nodal point N', and the corresponding image area is the circle on P_1P_3 as diameter. The total illuminated area is the circle on P_2P_4 as diameter. Suppose that the images formed at different obliquities are examined by a microscope of substantially greater aperture than is required to accommodate the full pencil through F' on the axis. If the microscope is directed to N' by the nodal slide, Fig. 3 illustrates that the aperture is only partly occupied by rays when P_1 is examined, and that many rays miss the objective. Long before P_2 is reached light has ceased to enter the microscope though a useful image is formed by the lens under test. In the lower part of the diagram the microscope axis has been adjusted so that the light passes symmetrically on the lens axis and for the edge of the illuminated field, as at P_4. At an intermediate position P_3 it is seen that useful light is again lost, and since the faults of adjustment at P_1 and P_3 lie in opposite directions it may be inferred that the translation of the point of intersection of the axes of the lens and microscope has been too great. As a guide to the selection of a suitable compromise diagrams may be constructed showing, for a series of selected axial intersection points, how the extreme edges of the pencil of light in the plane of the axis fall with respect to the objective aperture as the angle of obliquity is varied. The diagrams already given should not be used for this purpose, as owing to the difference of scale in the two directions the results would only hold for a lens of aperture $f/3$, a rapidity not yet nearly approached in a lens of the telephoto type. The results required may easily be calculated. Referring to Fig. 4, suppose that A is the point common to the two

axes and that P is the image point under examination. The microscope aperture plane is intersected by AP in C and by a ray BP from the edge of a lens aperture in D. AP meets this lens aperture in E. $BGH\mathcal{J}$ is parallel to the axis through B, G being in AP, H in the focal plane PF', and $\mathcal{J}$ on the perpendicular to AP through P. PC is of course the constant focussing distance of the microscope, and CD is the length it is desired to determine. Since $EB\mathcal{J}$ and $EP\mathcal{J}$ are right angles, the

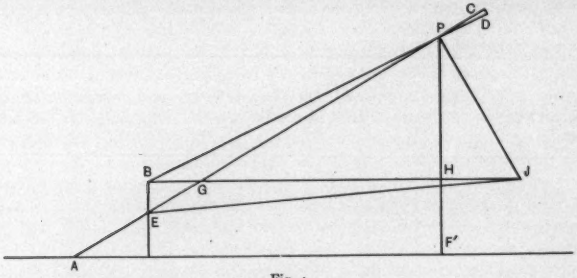


Fig. 4

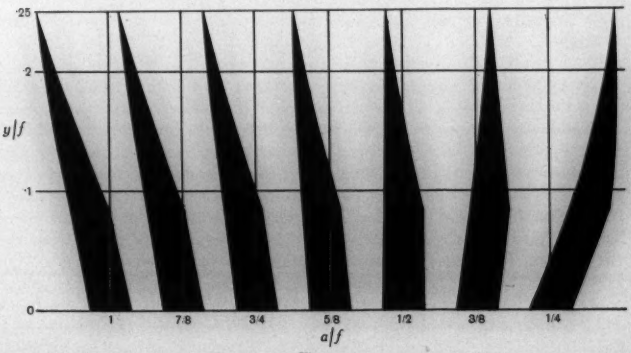


Fig. 5

angle $B\mathcal{J}E$ is equal to the angle BPE, that is to the angle DPC. Also the angles $EB\mathcal{J}$ and DCP are right angled, and therefore

$$\frac{CD}{CP} = \frac{BE}{B\mathcal{J}} = \frac{BE}{BH + H\mathcal{J}}.$$

Also $PH^2 = GH \,.\, H\mathcal{J}$ and thus from obviously similar triangles

$$\frac{CD}{CP} = \frac{ab - y(a - s)}{as + y(y - b)},$$

where $a = AF'$, $y = PF'$, $2b$ = diameter of the stop, and s is the distance of the stop from the image plane. For one stop b will carry a negative value, and for the other a positive value. Fig. 5 shows the results obtained by applying this formula to the lens described. Seven different positions have been taken for A, starting from the nodal point N' for the diagram on the extreme left, to a point between the last surface and the image plane on the right. The vertical line running through each diagram gives the centre of the microscope entrance pupil as a reference point. As the nodal slide is used to transfer attention from an axial point towards image points lying nearer the margins two effects can be read off the diagrams. In the lower part, where the angle made with the axis is smaller, the beam of light

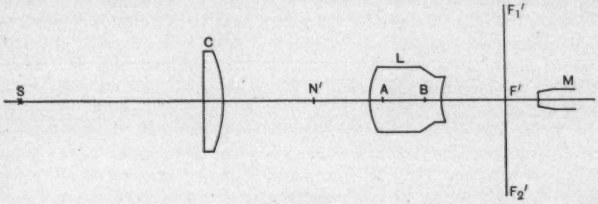


Fig. 6

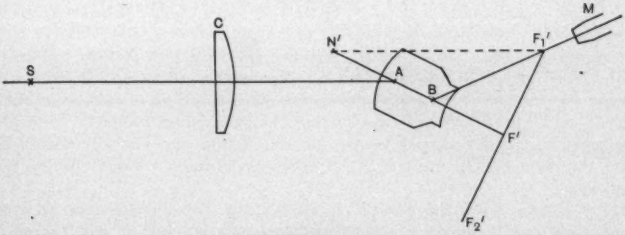


Fig. 7

is of practically constant width, in the upper parts where the figure tapers the light falls off continuously and vanishes at the point where the two sides of the diagram meet. The inclination of the outlines shows the extent to which the useful light tends to pass obliquely through the microscope. If vertical lines are added indicating the position of the edges of the microscope aperture the effects seen through the microscope are fully explained. In the present case the figure shows that the most satisfactory adjustment is when A lies a little toward the image side of the plane l_1, a being slightly under $f/2$.

The methods of applying in practice the principles that have been enunciated above may now be considered. In the first place it may be assumed that the test of the lens L (Figs. 6 and 7) is to be carried out on some form of optical bench

with the help of a collimator, having an artificial star S or other suitable object at the focus of its objective C. N' is the back nodal point of the lens and $F_1'F'F_2'$ represents the diagonal of the plate which the lens is supposed to cover, M being the objective of the observing microscope. Fig. 6 represents the arrangement in which the axial image of S at F', which corresponds to the central point of the plate, is under examination. Fig. 7, on the other hand, shows the relative arrangement of the apparatus when the point F_1' at one extremity of the field is being examined under the "ideal" conditions suggested above. The axis of the collimator is directed towards a point A behind the front surface of the lens and the microscope axis is directed towards a point B in front of the back surface, $N'F_1'$ being parallel to the axis of the collimator.

Now the ordinary photographic lens testing bench is not capable of giving this second arrangement automatically. It may therefore be of interest to discuss the alternative forms of the necessary modifications to be introduced. It is obvious that the most convenient axes about which to rotate the lens are those passing through N', A, and B.

(1) *Rotation about axis through* N'. This is the ordinary nodal slide arrangement, but in order to obtain the desired conditions it is necessary to provide the following additional movements. The collimator must be moved sideways relative to the bench in such a way that its axis remains parallel to the bench axis and passes through the point A. The axis of the microscope M must rotate about the point B. These movements could be obtained by the addition of suitable linkage systems to the ordinary nodal slide bench*. As the position of the nodal point N' is determined by experiment, some mechanism for adjusting the position of the lens relative to the axis of rotation over a considerable range would be necessary. If, instead of using a collimator, a distant object were employed, the only addition required would be the provision for rotating the microscope about the point B. It should be noted that it is necessary to maintain the distance between the microscope and the point on the plane $F_1'F'F_2'$ under examination constant during the rotation.

(2) *Rotation about axis through* A. In this case the collimator remains fixed and means must be provided for rotating the microscope round an axis through B and for maintaining the direction $N'F_1'$ parallel to the bench axis. This again involves the addition of suitable linkages, together with means for adjusting the position of the lens relative to the point N'.

(3) *Rotation about axis through* B. This arrangement is somewhat similar to that of (1) but is rather more complicated. It involves a parallel sideways displacement of the collimator and a rotation of the microscope about the axis through B, in addition to the maintenance of $N'F_1'$ parallel to the bench axis.

There are of course other possible methods of changing automatically from the arrangement of the apparatus as shown in Fig. 6 to that shown in Fig. 7, but they necessitate the adoption of rather more complicated mechanisms. Of the three

* The usual type of nodal slide bench does not lend itself readily to the mere addition of such systems of linkages, so that the bench would require to be re-designed.

methods outlined (2) is probably the simplest, unless one dispenses with the collimator and uses some distant object, in which case (1) is rather simpler.

When examining any particular lens on the ordinary nodal slide bench it is a very simple matter to determine whether proper use is being made of the apertures of the collimator and microscope objectives at all parts of the field. This can be done by removing the eyepiece of the observing microscope or by examining the Ramsden circle, either with the aid of a magnifying glass or with the eye held well behind the eyepiece. If the illuminated patch is approximately symmetrical with the microscope tube at all parts of the field, it is evident that the lens can be tested efficiently on the bench. If, however, the patch is not symmetrical, it is essential to employ such modifications of the bench as have been already suggested. It may be noted that an examination of the illuminated patch also gives the observer an idea as to which microscope objective should be employed in order to utilise its aperture most efficiently.

An alternative method to the nodal slide arrangement is possible where a suitable distant scale, graduated in angular measure, is available. The lens is set up with its axis parallel to that of the bench, and the observing microscope, mounted on an arm at right angles to the bench axis, is focussed on the centre of the field. The microscope is then moved along the cross-arm and at the same time rotated about an axis lying in its focal plane in such a way that its own axis passes through the point *B*. Thus different parts of the field may be examined and measurements of astigmatism, curvature of the field, distortion, etc. made. The disadvantage of this method is that it is very much more laborious than a nodal slide method in which the various movements are carried out automatically.

The authors hope in the near future to design a nodal slide bench which will be free from the limitations of the usual type of bench and will enable one to test telephoto lenses with as great ease as it is now possible to test ordinary types of photographic lenses.

DISCUSSION

Dr L. C. Martin (communicated): The promised new design for a nodal slide bench will be awaited with considerable interest, as it is clear that the only reason for retaining the nodal point rotations in testing benches has been the simplicity of the mechanism required for the continuous observation of the correct image, in spite of the fact that a large and expensive collimator lens is rendered necessary.

I hope that in consideraing the new design the authors will bear the following points in mind.

(1) For all ordinary work the instrument should be quick and easy to use. It is a serious disadvantage if weighty fittings have to be moved and adjusted. For pure testing work the bench is always in competition with the direct test of taking a photograph with the lens. Darwin's bench (*Proc. Roy. Soc.* 52 (1892), 403) has

the great advantages of simplicity and lightness for ordinary purposes. (2) When any measurements of image displacement perpendicular to the axis are necessary (in focal length and distortion tests for example) the use of linkage movements is apt to cause error.

The problems raised in these measurements are analogous to those in testing binoculars, but a far greater accuracy is called for. In securing the parallel displacement movement the use of a pentagonal prism might be helpful. From experience in the construction of "parallel" sliding movements, I think the requisite accuracy might be difficult to obtain by purely mechanical means.

Mr J. Guild, in referring to Dr Martin's remarks, said that it was surprisingly difficult to find a pentagonal prism which would behave exactly in accordance with theory when used for purposes of this kind. This was due to slight departures from homogeneity and flatness.

Mr F. Twyman said that in the design of the camera lens interferometer which he had recently described (*Trans. Opt. Soc.* 22 (1920–21), 174) he had been confronted with the problem dealt with in this paper. The difficulty could be overcome in the interferometer by using more nearly hemispherical mirrors, provided these could be made and tested to a sufficiently high degree of accuracy. He thought that the problem of constructing suitable linkage systems, such as the authors had suggested, must be tackled.

Authors' reply (partly communicated): The possibility of obtaining a suitable parallel motion by utilising a pentagonal prism was thought of, but was not considered to be of any great value.

Where possible, it is better to use a convenient distant object than a collimator. As pointed out in the paper, the collimator lens requires to have a much larger aperture than that of the camera lens under test, and in any case the aberrations of the collimator lens introduce difficulties.

The direct method of testing a camera lens by taking a photograph is of very little real practical value where a quantitative estimate of the aberrations is required. It may be of some use in the case of a good lens, but even then the micrometric measurement of the plate takes a considerable time. In the case of a poor lens results may be obtained by taking a series of photographs in different positions relative to the lens. If one is prepared to take so much trouble, the time would probably be more profitably spent in using the Hartmann test. From all points of view a nodal slide method is by far the most satisfactory.

A NON-POLARISING SPECTROPHOTOMETER

By A. J. BULL, M.Sc., F.Inst.P.

MS. received, 21st December, 1921. Read and discussed, 9th March, 1922.

ABSTRACT. The paper deals with a form of spectrophotometer in which uniform monochromatic patches of colour are compared, instead of the more usual arrangement of two portions of a spectrum. A spectrum is arranged so that the upper half has undergone selective absorption by the material under test, and a narrow region of the spectrum is selected by a slit. A split lens then forms two images of the dispersing prism face. These images are brought into juxtaposition by means of a rhomb-like prism with slightly unequal angles, which corrects the divergence of the two beams produced by the split lens. Photometric balance is secured by the partial closure of the lower portion of the selecting slit.

In several forms of spectrophotometer the light from the specimen under examination and the comparison white light are dispersed together so that a spectrum is obtained in which either the upper or the lower part has undergone selective absorption. Portions of this spectrum are then selected in succession by a diaphragm, or adjustable shutter, for examination, and the brighter half of the spectrum resulting from the dispersion of the white light is reduced in intensity until it balances the other half of the spectrum. This is the case with the Glan and the Hüfner spectrophotometers. In the Koenig-Martens instrument, however, the aperture of the telescope objective is viewed through an aperture placed in the spectrum, and is therefore illuminated with more or less monochromatic light. J. R. Milne* has also used a monochromatic field in a spectrophotometer.

One may consider the Hüfner as a typical example. In this instrument the comparison white light is plane polarised, while that under test passes above the polarising nicol. After dispersion, but while still parallel, both beams enter the analysing nicol. Therefore as this is rotated the light under test should be unaffected, but the comparison light is quenched according to the square of the cosine of the angle of rotation of the analyser, and by this means the two beams may be balanced. To make measurements, portions of the spectrum are selected for examination by an adjustable slit. In order to have a convenient field of view in which to secure a balance, the slit must be wide; but then a considerable range of the spectrum is viewed at one time, and it is impossible to secure measurements where the change of absorption is rapid, or in the middle of an absorption band. For this reason it is often necessary to narrow the slit, and then one is comparing two long rectangles of light placed end to end, the worst possible arrangement for securing accuracy of balance. Another defect of this particular instrument is that, as the analyser is rotated, the spectrum moves slightly across the field, so that the range of wave lengths being examined may not be accurately known.

* *Proceedings of the Optical Convention* (1905), p. 178.

In instruments where polarised light is used for adjusting the comparison beam to equality with the beam under examination, it is usual to assume that the intensity of the beam is proportional to the simple function of the rotation of the analysing nicol applicable to the particular instrument, although the difficulty of securing total extinction with crossed nicols throws some doubt upon the validity of the assumption that this law is the only controlling factor, especially as regards the higher angles.

The writer has experienced the further difficulty of having cleavage cracks develop slowly in the calcite of the nicol prism.

Königsberger's microphotometer has been applied to spectrophotometric work with a monochromatic illuminator, and the objections above described as to the difficulty of securing a photometric balance do not apply to it.

As a result of these difficulties, the attempt was made to design an instrument which would (1) bring the images to be compared into suitable juxtaposition, (2) arrange that the comparison is made between homogeneous patches of colour consisting of a very short range of wave lengths, and (3) avoid the use of nicol prisms for adjusting the comparison beam.

Description

In designing this instrument it was found convenient to start with the collimator and prism of a Hilger wave length spectroscope which gives a deviation of 90° for the region of the spectrum under examination. The instrument is shown in plan and elevation in Fig. 1, where *A* is the vertical slit of the collimator, *B* the colli-

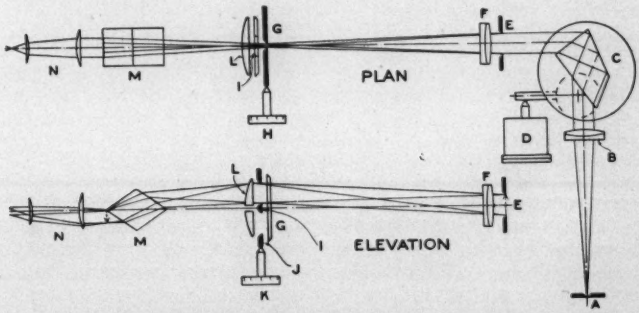


Fig. 1. Plan of instrument together with elevation of telescope portion.

A, collimator slit: *B*, collimating lens: *C*, dispersing prism: *D*, wave length drum: *E*, rectangular aperture: *F*, telescope lens: *G*, compound slit: *H*, micrometer screw for adjusting *G*: *I*, horizontal plate dividing *G* into an upper and lower portion: *J*, sliding leaf to close the lower part of *G*: *K*, micrometer screw for measuring the amount of movement of *J* over the lower part of *G*: *L*, split lens: *M*, rhomb: *N*, eyepiece.

mating lens with a focal length of 290 mm., and *C* the dispersing prism with its wave length indicating drum, *D*. The telescope objective *F* has a focal length of 700 mm. and has placed in front of it a rectangular aperture *E*, which measures 18 mm. horizontally, and 13 mm. vertically. A compound slit *G* is placed in the principal focus of *F*; this slit is adjustable in width by the screw *H*, and serves to select the short region of the spectrum in which examination is to be made. This slit has a total length of 25 mm., but the middle 5 mm. is covered by a horizontal plate *I*, thus dividing it into an upper and a lower slit each 10 mm. long. The lower slit is provided with a sliding leaf *J*, actuated by a micrometer screw *K*, so that this part of it can be closed from below upwards, thus reducing its length, and the amount of shortening can be accurately known. Immediately behind this compound slit is placed a split lens *L*, of 280 mm. focal length. The two components into which this lens is divided are separated vertically so that two images of the aperture *E* are formed by it. A glass rhomb, *M*, is placed with its second edge in the plane of the images of the aperture *E*, formed by the split lens, and this plane is viewed by a positive eyepiece *N*. The complete instrument is shown in Fig. 2.

The body whose selective absorption is to be examined is so arranged that light which has passed through it, or has been reflected from its surface, enters the lower portion of the collimator slit, while the upper portion receives the comparison beam. Both of these beams illuminate the whole of the collimating lens and are dispersed together by the prism. The telescope focusses the spectrum in a plane in which is placed the compound slit. This slit therefore passes a short range of wave length, determined by its width, the width of the collimator slit, and the dispersion. In order to ascertain the extent of this range, known bright lines are moved by the wave length drum from edge to edge of the slit when this is 0·5 mm., or any other convenient width, and a curve obtained, from which the values of this range for any part of the spectrum and any width of the compound slit can be found. A further correction has to be made for the confusion arising from the width of the collimator slit. The whole of the top 10 mm. of the slit is illuminated by light from the substance under examination, while the comparison beam passes through the adjustable lower slit. The split lens is shown as a simple lens in Fig. 1, but in the final form of the instrument it was made of two similar achromats. It forms two images of the aperture *E*, which are brought into juxtaposition by the rhomb *M*. These images are composed of the practically monochromatic light which has passed through the compound slit, and as the light in the plane of *E* is not yet dispersed, these images are of uniform hue. Various devices for bringing these two images into suitable juxtaposition so as to secure good photometric balance were tried, and the one found to be best consists of a glass rhomb with sides not quite parallel, the angles at the opposite ends being 60° and 58°. The smaller angle is placed in the focus of the eyepiece, and is polished to a fine edge. It acts like a Hüfner rhomb in bringing the two beams into contact by cutting off portions of each by total reflection. The slight difference in the end angles is to correct the divergence of the two beams caused by the decentering of the split lens components. The field can be limited by a circular or square diaphragm, and

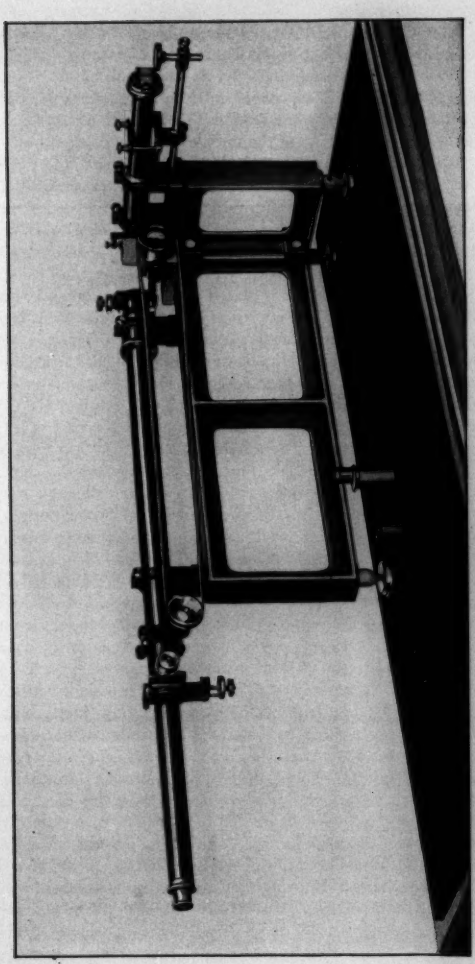


Fig. 2. General view of instrument.

is divided horizontally by the fine edge of the rhomb. The lower part of the field is then illuminated by the light under test, and the upper part by the comparison beam, and balance is obtained by adjusting the micrometer screw *K*. The reading of this screw therefore measures the transmission or reflection.

The accuracy of the measurements thus depends upon the illumination being uniform, not merely over the slits, but also over the aperture *E*, and is furthermore dependent upon the workmanship of the compound slit, the jaws of which must be accurately parallel in order that the assumption may be made that the intensity of the comparison beam is proportional to the length of the lower slit. The requisite accuracy of workmanship has been well realised by Messrs Adam Hilger, Ltd., the makers.

The uniformity of illumination is a difficult condition to fulfil, and various methods often employed to secure this, such for instance as reflection from a matt white surface and diffusion through a scattering medium, have proved none too satisfactory both in respect to the quantity of light and its uniformity. It therefore occurred to the author that use might be made of the principle employed in the "Kugel" photometer, utilising the internal reflections in a matt white hollow sphere to secure the requisite intensity and uniformity. By constructing such a spherical cavity within a block of pure white plaster and focussing the illuminant upon a small aperture in its side, a second aperture of small area at right angles to the first may be employed as a source of illumination for the slit. This device, in connection with the instrument now being described, and also in several other investigations such as the complete mixing of coloured lights, has proved to be entirely successful.

In order to facilitate working with the instrument, the optical system beyond the compound slit is made into a hinged eyepiece which turns back over the top of the telescope so that the compound slit can be examined directly, in order to ascertain before any measurements are made that the whole of the top upper slit is filled by light from the specimen, and the lower slit by light from the comparison white (Fig. 3).

In using the instrument it is important that the compound slit *G* be parallel to the collimator slit. This can be readily checked by noting that the image of the sodium lines is parallel to the jaws of *G*, when it is seen in the opening between them. A test for the uniformity of the illumination is to note the portion of the slit *A* which sends light to the image seen in the lower part of the field, then place a wire horizontally over this part of the slit, and adjust the photometric balance with *K*. On then moving the wire up and down over the part of the collimator slit under observation, keeping it horizontal while doing so, there should be no flicker nor disturbance of the photometric balance.

The instrument has proved to be quite efficient, but at the same time there are points where improvement might be effected, such as the slight confusion resulting from the curvature of the lines in the prism spectrum and the fact that no contrast design has been introduced into the field of view. The curvature of the spectrum results in a slightly greater range of wave length being included in the final patch

of colour than would be the case if this curvature did not occur, and when measurements of a low percentage reflection are being made in the yellow there is in consequence a slight difference in colour between the two halves of the field.

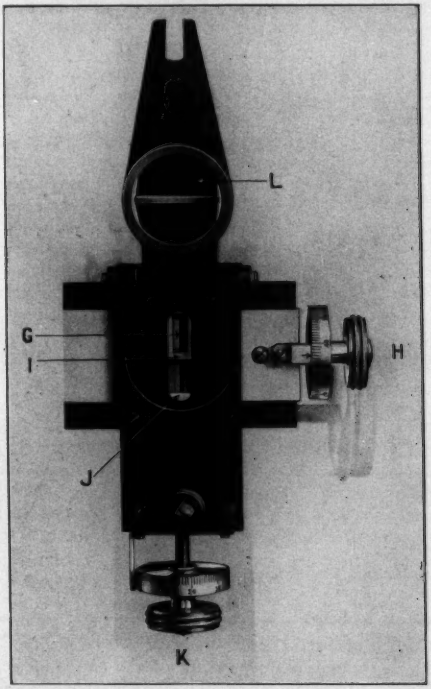


Fig. 3. Details of compound slit showing split lens turned back out of position.

Designed originally for measuring the selective absorption curves of printing inks, this instrument is peculiarly suitable for the measurement of pleochroic colours of minerals, since it does not employ polarised light for purposes of securing balance.

DISCUSSION

The President (Sir F. Dyson) asked to what special researches the author had applied his instrument and what was the degree of accuracy which he obtained.

Mr Bull, in reply, said that the bulk of his work had been with printing inks, especially those used in three colour printing. He had also employed his instrument in measuring the colours of minerals used for jewels, many of which are pleochroic.

It was difficult to state the accuracy obtained, as it varied with the state of the observer's eye and with the wave length. In the yellow and green he had been able to repeat readings to an accuracy of about 1 in 250, while for $\lambda 4200$ the accuracy was about 5 to 10 per cent.

Dr R. S. Clay suggested that the slit should be shaped to fit the curvature of the spectrum in the region of maximum sensitivity, that is, in the yellow.

He thought that it would be an improvement if the instrument could be provided with a triple field of view.

Mr J. Guild said that in his experience of various types of spectrophotometer he was convinced that improvements in the design of such instruments were badly needed, but he did not think the author had quite solved the problem.

The light given by the method of illumination employed would be entirely inadequate for measuring low transmission values. Transmissions less than 1 per cent. were often met: in such cases it was necessary to use an image of the source directly focussed on the slit. It was possible to obtain the requisite uniformity of field by employing a suitable condensing lens.

He agreed with the desirability of eliminating polarising devices, not so much on account of any inherent unreliability of nicol prisms, but because it was important to obtain results from instruments embodying different principles.

In the author's instrument inaccurate results would be obtained in measuring low transmission values, for when the slit is short diffraction effects vitiate the proportionality between area of slit and brightness of field and it would not be easy to make adequate corrections.

The difficulty of using polarising spectrophotometers with pleochroic minerals could be overcome by a simple device which he described in his paper on "The Photometry of Optical Instruments."

He agreed with Dr Clay's suggestion as to the use of a curved slit, but pointed out that another region of maximum sensitivity existed in the blue-green, and the curvature there would not be the same as in the yellow.

Dr L. C. Martin in a communication also raised the question of diffraction effects when the length of the lower slit is extremely small.

Dr J. S. Anderson said that the author, in reply to the President's remarks, had only dealt with the accuracy of repetition. He would like to know whether Mr Bull had checked any of his results by other means, such as rotating sectors.

Mr T. Smith pointed out that in most spectrophotometers inequality of illumination could be compensated by making measurements with the specimen first in one beam and then in the other; this could not be done in the present instrument.

He asked if the image of the slit *G* was sufficiently small to enable all the light to enter the pupil of the observer's eye.

The author, in reply, agreed that diffraction would cause errors with transmission values below 1 per cent., but the reflections of printing inks were rarely as low as 2 per cent. and this possible source of error did not impair the usefulness of the instrument for the work for which it was designed.

He had not checked his results by other optical means, but by measurements of multiple printing and variable columns of absorbing media and the results were consistent.

The whole of the light passed by the slit *G* goes to the formation of the final image and enters the pupil of the observer's eye.

THE PHOTOMETRY OF OPTICAL INSTRUMENTS.

By J. GUILD, A.R.C.Sc., F.R.A.S., F.Inst.P.

(Optics Department, National Physical Laboratory.)

MS. received, 23rd December, 1921. *Read and discussed, 9th March*, 1922.

ABSTRACT. The use of a portable surface-brightness photometer of the polarisation type for the measurement of the fraction of the incident light transmitted by an optical instrument or reflected by a mirror or system of mirrors, and for the measurement of the relative brightness of different parts of the field of view of an optical instrument, is described. The instrument employed is a Wanner optical pyrometer (Cambridge and Paul Inst. Co.) with some modifications to render it suitable for such work. An addition to the instrument is described which removes the danger of error due to polarisation of the incident light.

I. Introductory

The measurement of the percentage of light transmitted by optical instruments, particularly those of relatively complex design, is one of considerable importance. While it is easy to calculate the light loss due to absorption and reflection in the case of simple instruments, it is by no means easy to do so with any precision in the case of those with a large number of optical units without much more precise data than is sometimes available. It is often, therefore, necessary to determine the loss experimentally in order to compare the performance of instruments of similar function but of different design.

The method hitherto used at the laboratory for small instruments, such as binoculars, involves the adjustment of the instrument in relation to a photometer bench, and is unsuitable for instruments such as rangefinders and periscopes because of their size and non-portability, and, also, on account of the lateral separation of their entrance and exit pupils. The method of measurement to be described was evolved primarily for instruments of this type. It has, however, proved to be of considerable general utility for instruments of any size or type, the same apparatus serving not only to measure the percentage light transmission, but also the distribution of brightness in the field of view.

The method depends on the well known principle that the ratio of the intrinsic brightness* of an image to the intrinsic brightness of the object is independent of the magnification, depending simply on the loss of light by reflection and absorption within the instrument producing the image. If a luminous source of uniform intrinsic brightness be placed in the immediate neighbourhood of the exit pupil of a telescope system an image of this is formed at the entrance pupil (objective). The intrinsic brightness of this image divided by that of the source gives at once the proportion of light transmitted by the instrument.

* I.e., the light emitted per unit solid angle by unit area of the surface.

The necessary equipment therefore consists of a suitable light source of which the intrinsic brightness is uniform over an appreciable area, and an instrument for measuring intrinsic brightness.

II. Source of Light

As regards the source of light it is necessary to employ an illuminated diffusing screen, as there is no source suitable for use directly. After considerable experiment with various devices the arrangement shown in Fig. 1 was found to be very convenient for most purposes. *B* is a cubical lantern of wood or metal in which there are four 30 watt metal filament lamps (only two of which are shown in the diagram). *F* is a conical funnel at the front end of which is mounted the diffusing screen *S*. The interior of the lantern and funnel is painted with a matt white paint and this was found to ensure a sensibly uniform illumination all over the screen *S*. The nature of the screen itself is of some importance. Ground glass is quite unsatisfactory as the intrinsic brightness falls off very rapidly in directions inclined to the normal. The material actually used was Ilford Diffusing Medium. This screen is almost ideal for the purpose, and, when illuminated by a lantern of the type described, is equivalent, within a wide range of angle from the normal, to a self-luminous source of sensibly uniform brightness. Its diameter, as actually constructed, is about two inches. For some purposes a still larger source is desirable. It was found that a sheet of the diffusing medium of half plate size mounted in the long side of a whitewashed box, 40 cm. long, 17 cm. high and 13 cm. broad, containing a row of five incandescent lamps was of sensibly uniform brightness except near the edges.

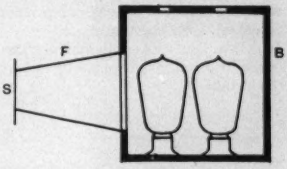


Fig. 1

III. The Photometer

The instruments commercially obtainable as surface-brightness photometers were not regarded as sufficiently accurate for the work, and a Wanner Optical Pyrometer, made by the Cambridge Scientific Instrument Co., which is essentially a surface-brightness photometer of the Koenig-Martens polarisation type, was adapted to the purpose. The essential parts of this instrument are shown in Fig. 2.

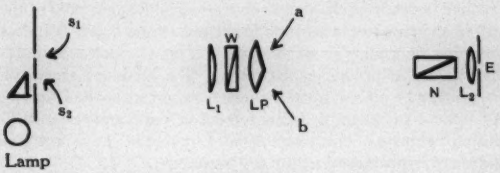


Fig. 2

s_1 and s_2 are two small circular holes in the focal plane of a collimating lens L_1; W is a Wollaston double image prism; LP is a combined lens and biprism; N is a nicol prism, which may be rotated about its axis; L_2 is a lens which enables the edge of the biprism to be sharply focussed by an eye placed behind a small circular aperture E.

Consider first light which enters the slit s_1: it is collimated at L_1 and passes through the double image prism, W, which splits it into two divergent beams polarised in mutually perpendicular directions. These, on reaching the biprism, are again subdivided, those rays which pass through the half a of the prism being deviated downwards (in the diagram) while those which pass through the half b are deviated upwards. The lenticular portion of LP focusses the various beams, so that four images of s_1 are formed in the plane of the aperture E. Of these two are due to light from side a of the biprism and two from side b, while of each pair one is polarised say, vertically, and the other horizontally.

If the aperture s_2 is also illuminated a similar set of four images of this is formed, displaced vertically (in the diagram) from the corresponding members of the other set. The arrangement of the parts is such that an image of s_1, due to light from side b of the biprism, coincides with the perpendicularly polarised image of s_2 from side a of the biprism, and also with the aperture E. The remaining three images of each set fall on the diaphragm and are not utilised. The eye placed behind E thus receives light from side a which comes from s_2 and is polarised, say, vertically, and receives light from b which comes from s_1 and is polarised horizontally. By rotating the nicol N the relative brightness of the two halves of the field may be adjusted to equality whatever the relative illumination of s_1 and s_2 may be; from the position of the nicol required for equality of brightness the relative intensity of the light entering s_1 and s_2 may be deduced.

In use one of the apertures is illuminated by a small four volt lamp and a right angled prism with matt surfaces, which serves as a standard of brightness.

When the instrument is directed towards a luminous surface rays from different points of it enter the aperture s_1 and reach corresponding points of the field b. Since all the rays intersect at s_1 the arrangement constitutes in effect a pinhole camera, and the brightness of the semicircular field b is proportional to the intrinsic brightness of the luminous surface, i.e., to the amount of light emitted per unit solid angle per unit area, and is independent of the distance of the surface from the instrument. In order that the semicircular field may be filled with light the luminous surface must subtend a greater angle at the pinhole than that subtended by the diameter of the field.

In the instrument as employed for optical pyrometry a deep red filter is mounted in front of the eyepiece, and a cap, in the centre of which there is a lens window, is provided at the other end. The effect of this lens is to focus fairly distant objects in the plane of LP, so that the particular part of a furnace on which the instrument is sighted may be seen distinctly. The circular scale for measuring the rotation of the analysing nicol only extends over a quadrantal arc, the temperature scale being engraved on the opposite quadrant.

In modifying the instrument for the present purpose the red filter and lens window were removed; the scale ring was cleaned and a finely divided scale extending round the complete circle was engraved. The latter alteration enables readings of both index marks (180° apart) to be taken, and settings to be made in the four positions of the nicol in which a brightness balance may be obtained. The mean of such observations was found to be correct when check measurements were made on rotating sectors or filters with known transmission coefficients, whereas readings taken with only one index arm at a time, or for one position of the nicol only, were usually in error.

The instrument as thus modified makes a very serviceable surface-brightness photometer, suitable for many purposes. There is, however, a possibility of more or less serious error arising in circumstances where the light entering the instrument is appreciably polarised, as in the case, for instance, where the light has suffered oblique reflections or refractions. This is a drawback to the use of polarisation photometers, well known to the users of such instruments; but, as far as I am

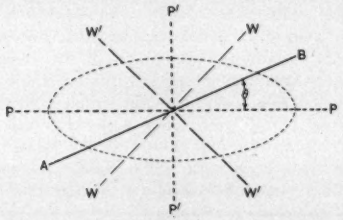


Fig. 3

aware, no attempt to overcome the difficulty has been made. By the simple device of mounting a quarter-wave plate in the front cover of the instrument, and orienting its principal directions to make an angle of 45° with the planes of polarisation of the Wollaston prism, the readings of the instrument may be made practically independent of the state of polarisation of the incident light.

Let AB, Fig. 3, represent a plane polarised vibration of amplitude a and period τ. The displacement of the vibrating particle from its position of rest is given, at time t, by $d = a \cos \frac{2\pi t}{\tau}$, t being measured from the moment at which the displacement is zero. The quarter-wave plate converts this vibration into an elliptic vibration with axes parallel to PP and $P'P'$, the principal directions of the plate.

The position of the vibrating particle in its elliptic orbit is given by

$$x = a \cos \left(\frac{2\pi t}{\tau}\right) \cos \theta,$$

$$y = a \cos \left(\frac{2\pi t}{\tau} - \frac{\pi}{2}\right) \sin \theta = a \sin \left(\frac{2\pi t}{\tau}\right) \sin \theta,$$

where x and y are the components of the displacement parallel to PP and $P'P'$ respectively and θ is the inclination of AB with respect to PP.

This elliptic vibration meets the Wollaston prism and is resolved into two plane polarised vibrations along WW and $W'W'$ at 45° to PP and $P'P'$. We are only concerned with one of these, say that along WW, and the displacement in this vibration is

$$\begin{aligned} d' &= x \cos 45^\circ + y \cos 45^\circ \\ &= a \cos 45^\circ \left\{ \cos\theta \cos \frac{2\pi t}{\tau} + \sin\theta \sin \frac{2\pi t}{\tau} \right\} \\ &= a \cos 45^\circ \cos\left(\frac{2\pi t}{\tau} - \theta \right). \end{aligned}$$

This represents a vibration of amplitude $a \cos 45^\circ$, that is, of intensity $a^2/2$, or half the intensity of the incident light. The orientation, θ, of the plane of polarisation of the incident beam merely affects the phase of the vibration resolved along WW and does not affect its intensity, which is exactly the same as if the incident light were unpolarised.

The adjustment of the quarter-wave plate may be made by trial, the instrument being sighted on a ground glass screen illuminated by polarised light and the plate rotated until the brightness of the screen, as measured on the instrument, is independent of the plane of polarisation.

It greatly facilitates matters, however, if the prism in front of the comparison pinhole can be removed so that both halves of the field are illuminated by light from the screen. The comparison side then receives the component of the elliptic vibration which is resolved along $W'W'$, and which, when the adjustment of the quarter-wave plate is correct, should be of equal brightness to that along WW for any orientation of the plane of polarisation of the incident light. The screen should first be illuminated with unpolarised light and the analysing nicol set to equalise the brightness of the two fields*. The illumination of the screen is now polarised by inserting a nicol prism in front of the source, or by other suitable means, so that the plane of polarisation is approximately parallel to WW or $W'W'$. In the absence of the quarter-wave plate one field would now be black and the other bright. On interposing the plate and rotating it into a suitable position the brightness of the two fields may be brought to equality, and will then remain in equality whatever the orientation of the vibrations of the incident light.

In order to facilitate this adjustment the comparison prism of the laboratory instrument was mounted on a sliding fitting so that it could be pushed into place or pulled out of the way by means of a short rod projecting through the side of the cap. This is necessary, since, if the cap has to be removed after adjustment of the plate in order to put the prism back in position, it is impossible to ensure that the adjustment will be correct after the cap is replaced.

In Fig. 4 two views of the instrument are shown for the information of those who may not be familiar with the Cambridge Pyrometer. In the lower view the

* The reading will be in the neighbourhood of 45° if the scale is in proper adjustment.

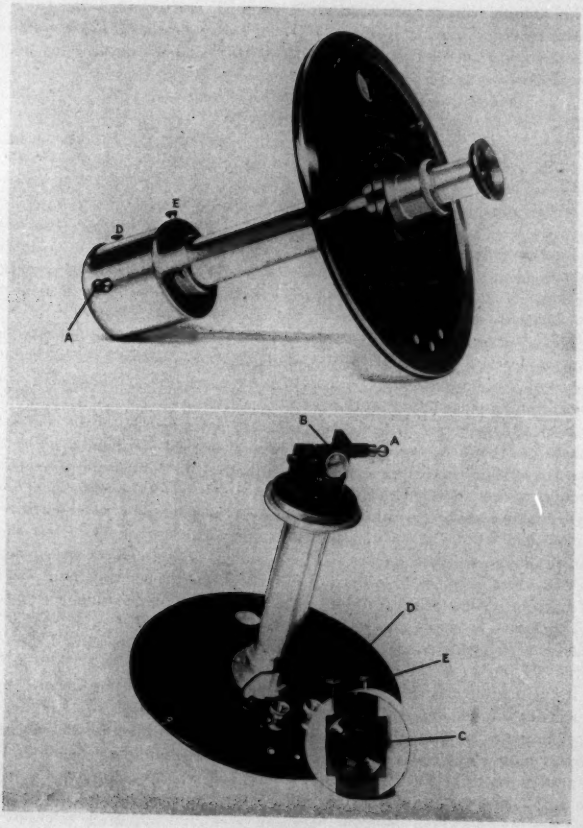


Fig. 4

cap is removed and shown separately. *A* is the projecting rod by means of which the comparison prism, *B*, may be withdrawn from its usual position in front of the second pinhole. The knob in which *A* terminates may be unscrewed to permit of the cap being removed or replaced. *C* is the rotatable mount for the quarter-wave plate, which, when adjusted, is clamped by a tangent clamp *D*. *E* is a locking screw to prevent accidental rotation of the cap.

The phase retardation of a crystal varies, unfortunately, with wave length, so the quarter-wave plate has only the required properties for one colour. For other colours the brightness varies slightly as the plane of polarisation of the incident light is rotated. As a result, when white light is used, a colour difference is set up between the two halves of the field for certain orientations of the plane of polarisation. It is therefore necessary, if the degree of polarisation of the incident light is high, to employ a colour filter to render the light more nearly monochromatic. The centre of the spectral region transmitted by the filter should of course be about the wave length for which the retardation of the plate is exactly a quarter-wave length.

The plate used by the author is of selenite and was taken from a set belonging to an old microscope with polarising accessories. Its retardation is such that when a light green filter (Wratten No. 57) is mounted in front of the eyepiece, the luminosities of the two sides of the field remain equal to within two or three per cent. whatever the orientation of the plane of polarisation of the incident light, although a slight colour difference exists in certain orientations. This is for completely polarised light, so that for the partial degrees of polarisation usually met with in photometric work the instrument is satisfactorily corrected. The use of a coloured filter may appear a disadvantage, but it must be remembered that it is only really necessary with light which is quite appreciably polarised, and in such cases the instrument would not be usable at all without the quarter-wave plate attachment.

In any case, even with completely unpolarised light, it is convenient to use the light green filter, as it is not then necessary to adjust the comparison lamp and the lamp supplying the illumination of the diffusing screen to exact equality of colour. The transmission factor of most optical instruments for green light does not differ appreciably from the value for white light. If it does so the transmitted light is coloured in any case and the "white" transmission factor cannot be measured. It is quite reasonable in such cases to employ for comparison purposes the transmission factor for green light as this, being near the region of maximum spectral visibility, gives a fair value for the luminosity of the transmitted light in the ordinary use of the instrument.

IV. Measurements of Transmission Factor

The procedure in determining the loss of light in an optical instrument by means of the apparatus just described is very simple. Fig. 5 shows the application to an instrument of periscopic type. The illuminated diffusing screen, *S*, is placed

in front of the eyepiece, approximately in the plane of the exit pupil. The photometer is placed in front of the object glass or window and sighted approximately parallel to the optic axis. The brightness of the screen *as seen through the instrument* can then be measured in terms of the comparison light in the photometer head. The lantern and photometer, being both portable, are then moved into the relative positions of Fig. 5 (*a*) and a direct measurement of the brightness of the screen made.

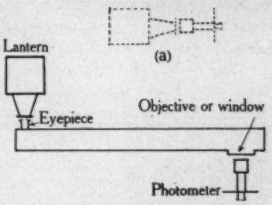


Fig. 5

The ratio of the brightness as measured through the instrument to the brightness as measured directly is the transmission fraction of the instrument for rays approximately parallel to the axis.

It is scarcely necessary to mention that the current through the lamps in the lantern and through the comparison lamp must be maintained constant during the whole determination. The ammeter supplied with the pyrometer is scarcely sensitive enough to give the constancy desirable for photometric work and a better instrument should be employed.

The method just described will be found very convenient for any telescopic instrument such as binoculars, periscopes, rangefinders, etc. provided the angular field of the instrument is not too small. Bearing in mind that the photometer is essentially a pinhole camera it will be clear that the angular dimensions of the beam of light entering the photometer are equal to the angular dimensions of the field of view of the instrument under test *as seen from the objective end*. If those dimensions are less than those of the semicircular field of the photometer, the latter is not filled, and, instead, a pinhole image of the field of the instrument is seen. The photometer can always be adjusted so that this image is partially eclipsed by the dividing line of the photometer field, and the brightness measurements are unaffected by the fact that the illuminated area does not occupy the whole of the available space. If the area is very small, however, the sensitivity of the measurements is very greatly reduced and it is difficult to obtain satisfactory results. The finite size of the pinhole also introduces error when the angular extent of the beam is very small.

This difficulty may be encountered in high power instruments with small fields. In such cases it is convenient to reverse the arrangement of Fig. 5, placing the source of light at the objective end and placing the photometer so that the pinhole is covered by the exit pupil. The angular aperture of the entering beam is now limited by the angular extent of the field of view as seen through the eyepiece and is greatly in excess of what is required to fill the field of the photometer.

If, as usually happens, the exit pupil is too near the eye lens for the pinhole to be placed at it, the photometer may be withdrawn some distance and an auxiliary lens of 10 or 12 dioptres employed to form a real image of the exit pupil on the pinhole. This auxiliary lens should also be interposed between the photometer and

the source when making the direct measurement of the brightness of the latter; its effect in reducing the intensity is then eliminated. If the instrument has a large entrance pupil the larger source, described earlier, should be employed, so that the effective exit pupil may be as large as possible.

V. Measurements of Variation of Field Brightness

The relative brightness at different parts of the field of view of an optical instrument has not, as far as I am aware, been experimentally determined. With a photometer of the type here described such measurements can be made with the greatest ease. The method is illustrated in Fig. 6. A luminous screen is placed at the exit pupil as in Fig. 5. It is necessary however that its brightness should be very much greater than is required for transmission measurements. On the other hand it need not be more than approximately uniform, and the most convenient

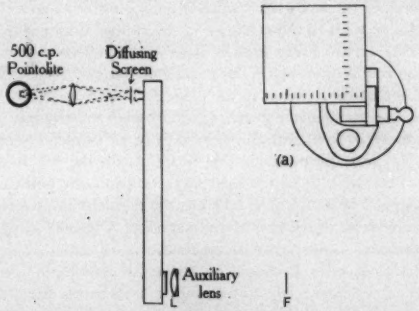


Fig. 6

arrangement is to focus an image of the luminous plate of a 500 candle power "Pointolite" lamp on a piece of diffusing medium situated in the plane of the exit pupil. The magnification should be such that the image is large enough to cover the whole of the area of the exit pupil with something to spare. The diffusing medium acts as a self luminous source emitting light in all directions, so that each point of the field receives light along every possible ray which can pass through the instrument.

An auxiliary object glass of diameter as great as, or greater than, the effective aperture of the instrument is mounted close in front of the latter at *L*, Fig. 6. If a sheet of diffusing medium is placed at *F*, in the focal plane of the auxiliary lens, an image of the field of the instrument under test will be formed on it. The total light falling on any part of this image will be proportional to the total light which reaches the retina from the corresponding part of the field when an observer looks through the instrument at a uniformly illuminated area, say a clear sky or an

extensive cloud, since, for the purposes of the test, we are merely using the instrument the wrong way round, replacing the image-forming parts of the eye by the lens L and the retina by the screen at F. The rays corresponding to any part of the field follow precisely the same path in either case, encountering the various stops, etc. in the same positions though in the reverse order.

Since the screen at F is almost perfectly diffusing, its *intrinsic brightness* at any point is proportional to the *total light* received per unit area of the screen at that point, so if we place the photometer close behind the screen and measure its brightness at different parts of the image we obtain measures of the relative amount of light reaching the retina from different parts of the field in the ordinary use of the instrument.

In order that the measurements may apply to regions as small as possible it is desirable that the pinhole should be close up to the screen. The most convenient arrangement is to remove the cap of the photometer* and attach the screen with plasticene or wax to the face plate containing the pinhole so as to cover the latter. Two scales may be marked off as shown in Fig. 6 (*a*), radiating from the pinhole. Needless to say, no mark of any kind is made at the point occupied by the pinhole which is at the intersection of the base lines of the two scales. The photometer can be placed so that the screen is in the plane F. The image of the field is focussed on the screen, and its diameter in the arbitrary scale units noted. By moving the photometer parallel to itself, either horizontally or vertically, any part of the image may be brought over the pinhole. The positions on the horizontal and vertical scales occupied by the edge of the field serve to locate the point occupied by the pinhole. It is generally sufficient to take measurements at a number of points along horizontal and vertical diameters, as the variation is usually approximately symmetrical with respect to the centre of the field.

The auxiliary objective L should be chosen, if possible, of such focal length that the image of the field formed at F, Fig. 6, is between one and two inches in diameter. If it is larger than this, the brightness of the photometer field is rather low, while if it is smaller there is difficulty in locating the precise part of the field to which a measurement corresponds.

It may not at once be obvious why the screen at F is required and why the photometer pinhole is not placed in the image plane without the interposition of such a screen. If this were done the quantity measured would be the intrinsic brightness of the field at the point occupied by the pinhole, that is the light per unit area per unit solid angle. The brightness of the field as observed in the use of the instrument is not proportional to the intrinsic brightness but to the total light reaching the retina from each unit area, and this depends not only on intrinsic brightness but on the angular aperture of the pencils entering the eye. As a matter of fact the intrinsic brightness varies very little over the field of most optical

* Note that measurements of the kind now under consideration are not affected by any polarisation in the incident light since they are purely relative. Not only so but the diffusing screen in front of the photometer almost completely depolarises light even if completely polarised to begin with. The quarter-wave plate attachment is therefore unnecessary.

instruments, the variations of field brightness being principally caused by variations of effective aperture due to the vignetting of oblique beams by the various stops.

If, however, the diffusing screen is interposed in front of the pinhole, its intrinsic brightness, which is what the photometer measures, is proportional not to the intrinsic brightness of the field of the instrument under test but to the total light reaching unit area of its image at F, or on the retina in ordinary use, that is, to the brightness of the field as observed in practice.

VI. Measurement of Reflecting Powers

The use of the instrument for measurements of the reflecting powers of mirror surfaces is very simple. The light source of Fig. 1 is first sighted directly and then by reflection in the mirror. This latter need not be plane, since magnification of the virtual image does not affect the measurements. We therefore have a very convenient method of measuring the reflectivity of curved surfaces, a process very difficult to perform by other means.

For such work of course the quarter-wave plate attachment is absolutely essential as most metallic surfaces polarise quite strongly. Silver is an exception, very little polarisation occurring in the reflection from silver or silvered mirrors.

The apparatus lends itself to measurement of the light reflected by a train of similar mirrors in series, since no precise adjustment of the various units is necessary, the photometer being directed towards the source as seen through the last mirror of the train.

Both lantern and photometer may be inclined downwards, so that measurements may be made on a horizontal surface such as mercury.

VII. Previous Work on Similar Lines

A polarisation photometer of similar type to that described here has been used to measure the transmission factor of optical instruments by F. E. Wright* in America. His work was done during the war, but an account of the method did not reach me until 1920, after the methods described here were in regular use in this laboratory. Wright did not use a comparison lamp in the photometer head, but mounted a periscopic prism system a few inches long in front of one of the pinholes. A source of considerable area is required as the procedure consists in sighting the photometer so that light which passes through the instrument enters one hole, exactly as in the author's method, while light which passes over the top of the instrument enters the other hole via the prisms. The arrangement is not suitable for use with periscopes, rangefinders, or any instrument in which there is considerable lateral separation of the entrance and exit pupils. The use of the comparison source in the photometer head as employed in the present method enables the transmission of any instrument whatever to be determined. The disadvantage that the separate light sources require careful voltage regulation is

* *Journ. Opt. Soc. Amer.* 2–3, 65, 1919.

more apparent than real, since such regulation has to be done in most photometric processes and requires no special equipment which is not likely to be found in most optical laboratories where photometric work is performed.

No previous attempt to render polarisation photometers free from errors due to polarisation in the incident light appears to be on record.

DISCUSSION

Mr A. J. Bull said that some time ago he had examined the polar curve of the diffusing medium which the author employed. He found that for a few degrees in the neighbourhood of the normal the medium acted as a self-luminous source.

Instructor-Commander T. Y. Baker thought that the method was very valuable, especially in dealing with cumbrous optical instruments. He had had a number of instruments tested by means of it and the results agreed very well with the theoretical values obtained by taking into consideration the total number of glass-air surfaces.

Mr H. S. Ryland described three methods which he had employed some years ago for comparing the transmission values of various prismatic binoculars. The first was a comparison method enabling a number of instruments to be checked quickly against two standards, an upper and a lower, forming a kind of optical limit gauge. In the second method photographs of the Ramsden discs of the instrument to be tested and of a standard were taken simultaneously and under the same conditions. The third was an autocollimation method of measuring the absolute transmission value of an instrument, part of the light after passing twice through the instrument being compared with the light which had not been transmitted.

The author, in reply, said he was interested to hear that Mr Bull had determined the polar curve of Ilford Diffusing Medium. The efficacy of the source described in the paper depended only partly on the diffusing properties of the screen and more on the fact that the light reached the screen from all directions owing to the high diffusive reflectivity of the white paint in the interior of the lantern and conical funnel. If a perfect paint were available, any kind of screen, e.g. a piece of ground glass, would be so illuminated as to scatter light uniformly in all directions and would be equivalent to a self-luminous source. The paint of course was not perfect, but when its deficiencies were compensated by using a really good diffusing screen, the result was a uniform emissivity over a very large angle from the normal.

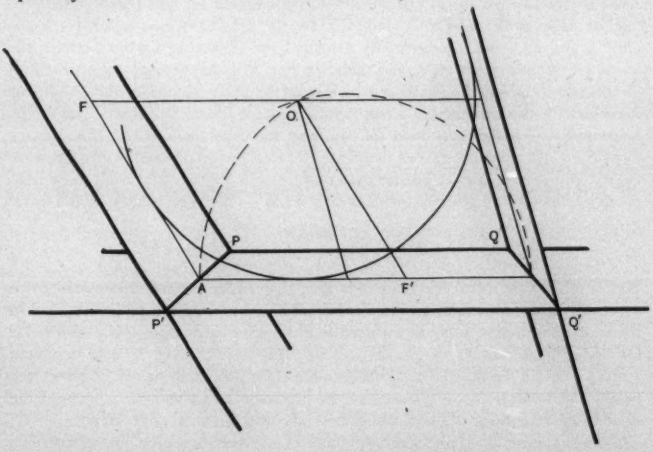
A PROJECTIVE TREATMENT OF THE SUBMARINE PERISCOPE.

By T. SMITH, M.A., F.Inst.P.

(Optics Department, National Physical Laboratory.)

MS. received, 16th January, 1922. Read, 9th March, 1922.

In a previous paper* communicated to this Society it has been shown that the general features of a periscope such as the number and the diameters of the lenses and their spacing may be discussed by considering the channel in the object space in which the incident rays to be utilised are contained, and that the details of the various lenses may be neglected for this purpose. The treatment then adopted depends on the invariant relation between lateral and longitudinal magnifications. It has since occurred to the writer that the events which occur in the instrument may be illustrated by homocentric projection, and that this affords a simple means of finding the relative advantages or disadvantages of various arrangements of the optical system.



Referring to the figure, let O be a point equidistant from two planes which intersect in the straight line PP'. Suppose that A is the foot of the perpendicular to PP' from O, and that FA, AF' lie each in one of the two planes, $OFAF'$ being

* "The Optical Three Apertures Problem," *Trans. Opt. Soc.* 23 (1921–22), 132.

a rhombus in the plane normal to PP'. If the two planes FPP', $F'PP'$ are regarded as axial sections of the object space and image space for a thin lens with F, F' as its principal foci and PP' for its coincident unit planes, any object point is connected to its image point by a straight line which passes through O. The angle which the planes make with one another is immaterial, and if the image plane is rotated about PP' the locus of O is a circle described about F as centre. Similarly if the image plane is fixed and the object plane is revolved about PP', O will move along a circular path with its centre at F'. If a second lens is introduced which meets the first image plane in QQ', the effect of the second refraction may be determined through the intermediary of the first image plane by projecting through a suitable point O' on to a third plane which contains the straight line QQ'. If the second plane is kept fixed and the first and third are rotated, O and O' describe coplanar circles when the two lenses have a common axis, and under suitable conditions these circles intersect, and projection from this common centre gives all the correspondences that exist. In some cases further projection from the same centre is possible, and the effect of many successive refractions may thus be exhibited in a particularly simple way.

If the optical system consists of a succession of separated lenses with equal diameters, and it is assumed that all rays which pass through the first two apertures must be transmitted by all later lenses, the conditions for real homocentric projection are satisfied. For if A_n and A_{n+1} are the vertices of two successive lenses and F_n and F_{n+1} the corresponding principal foci in the medium separating the lenses, A_n must precede A_{n+1}, and therefore $F_{n+1}F_n$ is less than $f_n + f_{n+1}$, the sum of the two focal lengths (both of which must be positive) one of the conditions necessary for the existence of a real point O distant f_n and f_{n+1} from F_n and F_{n+1} respectively. Another condition follows from the consideration that the distance from F' to QQ' must not exceed that from PP' to F', as otherwise the magnification at QQ' will be numerically greater than unity, that is the aperture will not be large enough to include all possible incident rays. Thus if t is the separation of the lenses

$$t \geqslant 2f_n,$$

or

$$F_nF_{n+1} = f_n + f_{n+1} - t \leqslant f_{n+1} - f_n.$$

By combining the two conditions it is seen that under the conditions laid down successive transformations may be illustrated by a homocentric projection, and in types of construction such as are used in submarine periscopes this result holds for any assigned number of transformations. It follows that the properties of such a periscope may be derived by considering a right polygonal prism, of which the height corresponds to the diameter of the periscope lenses and the lengths of the sides to the lengths of the sections of the body between successive lenses.

As a rule after a certain integral number of sections have been passed previous conditions are exactly reproduced, either with or without lateral inversion, and the corresponding prism is closed, and in the latter case is composed of two symmetrical halves. When the sections are of equal length the prism is built upon a regular polygon. Each side of the polygon is tangential to a circle with the point of pro-

jection for its centre, and the general result follows that if similar conditions are found after an even number of cells have been passed, the total length of the odd cells must be equal to that of the even cells. If on the other hand the number is odd the total length of the series of alternate cells which includes the extreme cells must exceed that of the remaining cells.

The main properties of the periscope may be readily found by considering the polygon alone, and simple constructions for the polygon are forthcoming when the lengths of the sides are given. For example, when there are three sides to an inversion of lengths l_1, l_2, l_3, a triangle may be constructed with its sides proportional to $\sqrt{l_1}, \sqrt{l_2}, \sqrt{l_3}$. If one of the sides is taken as the unit of length, the opposite vertex is the centre of the circle about which the hexagon is to be constructed. If it is preferred to calculate the diameter the formulae

$$d = \{2(l_1l_2 + l_2l_3 + l_3l_1) - l_1^2 - l_2^2 - l_3^2\}^{\frac{1}{2}} = \{L(L - 2l_1^{\frac{1}{2}})(L - 2l_2^{\frac{1}{2}})(L - 2l_3^{\frac{1}{2}})\}^{\frac{1}{2}}$$

may be used where $$L = l_1^{\frac{1}{2}} + l_2^{\frac{1}{2}} + l_3^{\frac{1}{2}}.$$

From the geometry of the figure it is evident that the powers of the lenses in a system of this kind may be represented by

$$l^{-1} \sin 2\alpha_1,\ l^{-1} \sin 2\alpha_2,\ l^{-1} \sin 2\alpha_3, \ldots l^{-1} \sin 2\alpha_n$$

and the separations by

$$l(\tan \alpha_1 + \tan \alpha_2),\ l(\tan \alpha_2 + \tan \alpha_3), \ldots l(\tan \alpha_{n-1} + \tan \alpha_n).$$

The four Gaussian constants of the combination are

$$\frac{\sin A}{l},\ \frac{\cos(A - \alpha_1)}{\cos \alpha_1},\ \frac{\cos(A - \alpha_n)}{\cos \alpha_n},\ -\frac{l \sin(A - \alpha_1 - \alpha_n)}{\cos \alpha_1 \cos \alpha_n},$$

where $$\tfrac{1}{2}A = \alpha_1 + \alpha_2 + \alpha_3 + \ldots + \alpha_n.$$

When $A = \pi$ the system is a telescope of power -1, and when $A = 2\pi$ the power is $+1$. In either case previous conditions recur if the earlier lenses and separations are repeated after a preliminary interval of $l(\tan \alpha_n + \tan \alpha_1)$. The length attained in a cycle is

$$2l \sum_1^n \tan \alpha_p,$$

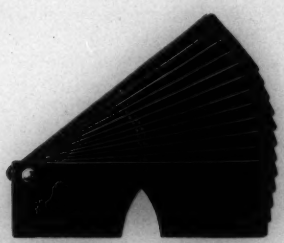
and the sum of the powers of the lenses is

$$l^{-1} \sum_1^n \sin 2\alpha_p.$$

The projective properties of this system may be utilised to obtain from a hollow prism, of which suitable parts are transparent and others opaque, in conjunction with an internal source of light such as a "Pointolite" lamp, the effective positions and sizes of the stops for a given plane, or alternatively if an eye is placed directly or by dioptric means at the centre, a suitable telescopic system for transmitting the light falling within a given channel is obtained by the adjustment of planes on a cylinder to project a plane sectional outline of the assigned channel. Such methods, or their trigonometrical equivalents, appear to offer a far more rapid solution of such problems than the means hitherto described.

FURTHER DISCUSSION ON DR FRENCH'S PAPER "THE INTEROCULAR DISTANCE"

MR H. L. TAYLOR: I am induced to offer a few remarks upon Dr French's paper read before the Optical Society in November last, because so many ophthalmic opticians will be unable to reconcile his conclusions with their measurements as recorded for the purpose of supplying spectacles, etc. Some years ago I took a large number of interpupillary measurements in many parts of England and Wales, and compared them with others, taken by competent observers in each district, for the purpose of verification. The methods mainly used were those of apposition with cross lines, an adjustable frame with carriers bearing small knobs, and a device known as the inter-visual-axis measure.



The first two methods are so well known that I do not purpose describing them, but the last is not so generally used. It consists of a series of flat scales (see figure) each being pierced by two pinholes, which are from 53 to 64 mm. apart, so that there are twelve scales, movable one upon another. Upon one side of each scale is a triangular aperture, whose apex comes midway between the pinholes. This serves primarily to admit the nose when the scale is used. It also has a further use (*Proc. Opt. Convention*, 1912). In using the device that scale is chosen which gives coincidence of the two circles seen when both eyes have vision through the pinholes. Coincidence is verified by tilting the scale, when the circles separate in the same vertical line. Such verification is most necessary for accuracy.

This is a true subjective test, but can only be used with intelligent persons. It measures the distance between the visual lines in the plane of lenses worn before the eyes. Such measurements rarely tally with those made objectively, which measure the distance between the centres of the pupils, because the pupillary line does not coincide with the visual line, but forms an angle with it of something like 5° in most cases, occasionally being 2° or 3° more. The visual line is on the nasal side.

I have used the Zeiss pupilometer employed by Dr French, but have found several sources of error, if we are concerned with the position of the visual axes and not merely points upon the pupil. Owing to the index line being about 10 inches from the eye it is often difficult for many adults, unless myopic, to focus it, and so it appears as blurred, and one has to estimate the centre of the pupil with the blurred line in front of it.

Usually there will be a difference of about 2 mm. between the findings with the Zeiss pupilometer and a subjective inter-axial measurement, and only in few cases have I found them alike. The average measurements given by Dr French I should consider are 2 mm. too high for general use. Allowance must be made for the type of man chosen, but taking a number of cases, and comparing height of person and width of head, the difference between Dr French's figures and my averages would be over 1·5 mm.

I found that figures varied with districts. In parts of N.E. and S.W. England they are high, the variations being apparently dependent upon stature and cranial characteristics. The measurements for Wales are below the average. The North Welshman is taller, has a longer skull, and is not so heavily pigmented as his bullet-headed southern brother, although the admixture of so many foreign elements in South Wales invalidates the findings, notably in areas like Pembrokeshire and Gower, where the large numbers of English-speaking people are mainly of Flemish descent.

Dr French: I have read with much interest Mr Taylor's remarks which clearly emphasise, I think, the desirability of obtaining measurements over the whole area of the country. If Mr Taylor would be so kind as to tabulate his results, I feel sure they would be a valuable addition to the data at present available.

If the mean result indicated by Mr Taylor is obtained from measurements of adults of both sexes, then his mean value is, I presume, 60, as compared with my mean of 62, and I think 60 is too low, although it may be a good enough mean in spectacle practice with average convergence of the eyes. The pupilometer with which my tests were taken gives results that are confirmed by experience with high power binocular instruments, for use with which, indeed, the apparatus was originally designed.

Mr Taylor does not give particulars of the minimum age of his subjects. It is essential that the minimum age of those included in the tests should be considered. In my results individuals of seventeen years and under were excluded. It will be evident that, if an appreciable number of undeveloped persons are included in the results, the average will be lowered.

Mr Taylor has employed three methods of measurement. The first two are not sufficiently described to enable me to understand them clearly, but I presume the second method is comparable with callipers, the points of which carry balls. This method was used by me to check the Zeiss pupilometer readings. The third method may be good enough for approximate spectacle work: I do not think it can provide results of much accuracy. It is intended for spectacle work, the holes being situated

in the plane of the glasses, whereas in the pupilometer the scale can be placed close to the cornea. Further, the notch for the nose is situated halfway between the two points whereas in the majority of individuals the nose is not symmetrically situated with respect to the pupils. The higher limit of 64 is small.

Reference has been made to the angle between the visual and axial lines. In view of the small distance of the mean principal point from the pupil, the error involved is comparatively small.

Mr Taylor objects to the Zeiss pupilometer for the reason that adults, unless myopic, have often difficulty in focussing the objects. Of 400 individuals tested, there were only two who had any difficulty in focussing the scale, which is situated at the normal distance of ten inches.

I should look forward with much interest to having an opportunity of analysing Mr Taylor's data, because to obtain a true mean very careful analysis of results is essential.

Mr Taylor: With reference to Dr French's reply to my criticisms, I have only considered males above 17 years, in order to comply with Dr French's conditions, but the ages would extend well above 50 years, whereas the ages given in the tables cease at that.

With the scale for subjective measurement the limit of error is ·5 mm., an amount easily appreciated in using the device. The notch has nothing to do with the inter-axial measurement, being intended for the estimation of the asymmetry of the eyes referred to, a matter dealt with elsewhere (as referred to).

My measurements for distance have always been taken against the sky, but I have taken great numbers for convergence, using a white screen at any desired distance, the figures obtained being valuable as confirming the distance test.

The reason why Dr French found only two individuals who could not focus the scale, is that his tables show no age of over 50 years.

REVIEWS

Die Anwendung des Mikroskops. By R. SCHMEHLIK. 108 pp. and 21 pp. of illustrations. (Berlin: Union Deutsche Verlagsgesellschaft, 1922.) Price (in Germany), 24 Marks.

To all British workers with the microscope who command a sufficient knowledge of German this little work can be warmly recommended, as, whether they are beginners or "old hands," they will find much interesting matter in it not readily available elsewhere. The author, a lecturer on microscopy and optics, having been unable under present circumstances in Germany to issue a voluminous book on the subject, has condensed his experiences, ranging over nearly every branch of microscopy, including micro-projection, photomicrography, micro-stereoscopy, micro-metallurgy, light filters, etc., into this little booklet of 108 pages. There is consequently a refreshing absence of too much "padding," which students of German scientific literature will appreciate.

The book is eminently a practical one, giving the reader precise instructions how to set up and manipulate his apparatus to obtain the results desired, together with the reasons, and the author being one of those who delight in setting up experiments by simple and improvised apparatus, we find many ingenious "dodges" and home-made arrangements described. He constantly has in mind the student who cannot afford the luxury of expensive accessories, as well as those who have a longer purse at their disposal.

It is not possible in a short review to discuss adequately the many points, theoretical and practical, that are dealt with, but attention may be drawn to a few of the experiments described, such as (1) the making visible of the paths of the diffracted beams from a grating by an arrangement in which a block of uranium glass, or a cube prepared of gelatine and fine starch grains is placed between the object and the objective, (2) comparative photographs showing increase of detail seen in metallic surfaces when polarised light is used instead of ordinary light, (3) projecting stereomicrographs on to a lantern screen and viewing them with a Galilean binocular, in which the tubes have been taken apart and fitted with an arrangement that they can be adjusted to different degrees of divergence, (4) an improvised apparatus in the nature of an Abbe Demonstration Microscope, and (5) series of photographs of a piece of muslin, showing the various false images obtained by blocking out specific spectra.

In perusing the book it should not be overlooked that there are points in which the German school of microscopists still differ from those of the English school. Thus we find it stated particularly in regard to photomicrography, that the image of the source of light should be projected on the iris diaphragm of the condenser, whereas we should always focus it on to the object itself, excepting in the case of low powers, where in certain circumstances it may be necessary to adopt the former method in order to obtain a sufficiently large illuminated field. It is the more curious that this method should still be recommended, inasmuch as the author himself points out how without due precautions a duplication of structure may arise in the image, and gives an excellent photograph of Navicula in which this result has been purposely arranged.

It must not, therefore, be assumed that all of the author's recommendations are quite safe, but sufficient has been said to show the interest of the book to English readers.

A somewhat novel and convenient feature is that the 21 plates containing 131 photographs and diagrams are bound separately so that they can be more easily referred to whilst reading the text of the book.

J. R.

The Theory of Modern Optical Instruments. By Dr ALEXANDER GLEICHEN. Translated from the German by H. H. EMSLEY, B.Sc., and W. SWAINE, B.Sc. *Second Edition*. Pp. xii + 365; figs. 241 + 2. (London: Published by H.M. Stationery Office, 1921.) Price 12*s*. 6*d*. net or 13*s*. 6*d*. post free.

The first edition of this useful compilation on optical instruments was reviewed in these *Transactions*, 20 (1918–19), 204. In the present edition a few further footnotes have been added and also two new appendices. The first appendix describes the optical system of a cystoscope made in this country, and the second, which occupies rather more than 20 pages, deals with photographic objectives including those of British origin. The appendix on the Barr and Stroud rangefinder which was printed in the first edition has been omitted with a view to its inclusion, together with descriptions of other instruments, in a separate volume. A note added since this volume was printed states that the proposal to issue this further volume has been abandoned as a firm of publishers have a book on modern optical instruments in preparation.

The errors which were noted in the review of the first edition have been corrected with the exception of figures 34 and 35. In an actual system of the type shown the images of the diaphragm would necessarily be crossed. The new matter should have been more carefully checked. On p. 70 in a discussion on chromatic aberration ν has been given a sign opposed to that in the section to which reference is made. A misprint occurs in the formula for first order spherical aberration on p. 74, and in the new section on photographic lenses, the value of which depends entirely on the reliability of the figures quoted, a number of errors are present. Unfortunately the authors have not been able to quote any particulars of several of the best known lenses figured in this section. Messrs W. Watson and Sons are omitted from the list of manufacturers of photographic lenses, and Messrs Taylor, Taylor and Hobson should have been mentioned as the makers of Cooke lenses.

T. S.